Final Exam

Allowed: Time – up to 3 hours, formula sheet (given) and calculator

Answer **all** questions from **Section A**, and **two** out of three questions from **Section B**.
Both sections are worth equal marks.

Section A

A1. Consider two equally intense co-propagating red laser beams in free space, with angular frequencies of ω_1 and ω_2, respectively, where $|\omega_1 - \omega_2| < 2\pi \times 1\,\text{GHz}$. Calculate the form of the irradiance (also called the intensity) that would be detected by a photodiode which detects frequencies from DC up to 1 GHz.

A2. Explain, using a few sentences, the information conveyed by equation 2.8 on your formula sheet, carefully explaining what each term represents.

A3. Determine expressions for the phase speed, and skin depth [where the intensity has dropped by a factor of $1/e$], of light once it has penetrated a metal surface with high conductivity.

A4. Calculate the linewidth, $\Delta\nu$ in MHz, of the transition of ^{20}Ne in a HeNe laser operating on the 632.8 nm transition, assuming the gas temperature is 400 K.

A5 For a typical laser transition with inhomogeneous broadening, determine an expression for the peak cross-section of the transition, $\sigma_0 \equiv \sigma(\nu_0)$, in terms of its linewidth $\Delta\nu$ and the spontaneous decay rate of the transition, A_{21}. The erbium fiber laser at an output wavelength of 1550 nm, and $n = 1.46$,is a three-level laser system, with almost all decays from the upper laser state falling directly to the lower laser state, with a lifetime of 10 ms. The linewidth of the transition at room temperature is 5 THz. What is the peak cross-section of this transition under these circumstances, σ_0? [to account for the refractive index, replace c by c/n]

Section B

B1.a) Apply the standard electromagnetic boundary conditions at a vacuum-conducting interface (with the conductivity of the material being σ, assumed non-magnetic) to show that at normal incidence the *reflection coefficient*, *r*, is given by:

$$\tilde{r} = \frac{1 - \dfrac{c\tilde{k}_T}{\omega}}{1 + \dfrac{c\tilde{k}_T}{\omega}} \quad ,$$

where $\tilde{k}_T$ is the transmitted complex wavevector in the conducting material.

b) Given this, for a very good conductor (with $\tilde{k} \approx \sqrt{\frac{\mu_0 \sigma \omega}{2}}(1+i)$) use the binomial approximation to show that the dependence of the *reflectivity*, *R*, on angular frequency is given by $R \approx 1 - 2\sqrt{\frac{2\omega\varepsilon_0}{\sigma}}$. Sketch the general form of this function against <u>incident wavelength</u>, λ.

B2. Consider a unidirectional ring laser where light travels from mirror 1 (99% reflective) to mirror 2 (99% reflective), through a gain medium, and towards the output coupler (95% reflective, 4% transmission) before being either transmitted out of the laser or reflected back to mirror 1. The gain medium is a 10 cm long Nd:YAG crystal with a saturation irradiance of 2300 W/cm^2 and the crystal is pumped at a rate that results in a small-signal gain coefficient of 0.05 cm^{-1}. Assuming steady state operation:

a) Make a sketch of the ring laser, and find the threshold gain coefficient of this cavity.

b) Find the irradiance of the laser field that exits from the output coupler.

c) What is the irradiance at each end of the gain medium inside the cavity? What are the corresponding values of the gain coefficient, γ, and how do these compare with the threshold gain coefficient? What is the name given to this phenomenon, where the gain coefficient reduces from its small-signal value?

B3. a) Explain, with the aid of one or more diagrams, the physical principle behind the *light emitting diode*. In three or four sentences, explain the information given in the diagram below:

{Figure 16.1-7 from Saleh and Teich, 2nd ed.}

b) Explain, with the help of one or more equations, the physical principle behind *non-linear optics*. Show mathematically how the two phenomena known as sum frequency mixing, and difference frequency mixing, arise. What requirement is placed on a crystal to allow such phenomena to occur?