Midterm: Tuesday Oct 29, 2024 Name:__________________

Allowed: Formula sheet (given), calculator, 1 hour 50 mins

PART 1 – answer questions **1-8** in your exam booklet and **then** use your scratch card:
One scratch = **100%**; two scratches = **33%**; three scratches = **25%** (part 1 total is 50%)
PART 2 – answer questions **9 & 10** in the exam booklet provided (each of these questions is worth 25%)

Qu's 1-4) A linearly polarized electromagnetic wave travels in the +*x*-direction in dry air. The amplitude of the electric field is $4.6\ \text{V/m}$, the frequency of the wave is 480 THz, and the precise direction of the polarization is unspecified (although you may assume that the initial phases of all electric and magnetic field components are zero).

1) With all values in SI units, which of the following expressions is the best scalar representation of the electric field of the wave?

A. $E = 4.6\cos(1.01\times10^{7}x - 3.02\times10^{15}t)$ **B.** $E = 4.6\cos(1.73\times10^{7}x - 5.20\times10^{12}t)$

C. $E = 9.2\cos(1.73\times10^{7}x - 5.20\times10^{12}t)$ **D.** $E = 9.2\cos(1.73\times10^{7}x + 3.02\times10^{12}t)$

E. $E = 4.6\cos(1.60\times10^{6}x + 3.02\times10^{12}t)$

2) Which of the following values is the peak *magnetic field* of this wave?

A. 15 nT **B.** 15 μT **C.** 12 nT **D.** 12 μT **E.** 24 nT

3) The wave enters a glass slab with its front surface perpendicular to the direction of travel, in the *yz*–plane. The speed of the wave within the glass is $1.65\times10^{8}\ \text{m/s}$. Which of the following forms could represent the transmitted electric field of the wave inside the glass, with amplitude E_{0t}?

A. $\vec{\mathbf{E}} = E_{0t}\cos(1.09\times10^{7}x + 3.02\times10^{12}t)\hat{\mathbf{y}}$ **B.** $\vec{\mathbf{E}} = E_{0t}\cos(1.73\times10^{7}x - 5.20\times10^{12}t)\hat{\mathbf{z}}$

C. $\vec{\mathbf{E}} = E_{0t}\cos(2.07\times10^{7}x - 3.02\times10^{12}t)\hat{\mathbf{y}}$ **D.** $\vec{\mathbf{E}} = E_{0t}\cos(1.83\times10^{7}x - 3.02\times10^{15}t)\hat{\mathbf{y}}$

E. $\vec{\mathbf{E}} = E_{0t}\cos(1.50\times10^{6}x + 5.20\times10^{12}t)\hat{\mathbf{z}}$

4) The slab of glass is now rotated about an axis parallel to the *z*-axis, so that the *front face* of the glass makes an angle with the *x*-axis of 60^{0}. By which rotation angle is the ray deflected from its original path?

A. 14° **B.** 16° **C.** 18° **D.** 30° **E.** 36°

5) The light emitted by an LED is centred at 670 nm and has a wavelength spread of 0.9 nm. Which of the following is the this spread given as a frequency spread?

A. 450 GHz **B.** 220 THz **C.** 730 GHz **D.** 600 GHz **E.** 330 THz

6) The intensity of light from the Sun striking a surface perpendicular to the Sun's rays is about $1350\ \mathrm{W/m^2}$. By approximating this light as a single harmonic wave, which of the following is the amplitude of the electric field for a wave with this intensity?

A. 450 V/m **B.** 1000 V/m **C.** 10 V/m **D.** 8.7 V/m **E.** 870 V/m

Qu's 7-8) A light beam has an electric field that takes the following form, in units of V/m:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},t) = 4\cos(kz-\omega t)\,\hat{\mathbf{x}} + 4\cos(kz-\omega t-\pi/2)\,\hat{\mathbf{y}}$$

7) Which of the following statements best describes this light beam?

A. Plane polarized light, with inclination of $+45°$ to the horizontal (so $45°$ counterclockwise from the horizontal when looking into the beam) travelling in the $-\hat{\mathbf{z}}$ direction.
B. Left circular polarized (electric field rotating counterclockwise when looking into the beam), travelling in the $+\hat{\mathbf{z}}$ direction
C. Plane polarized light, with inclination of $-45°$ to the horizontal (so $45°$ clockwise from the horizontal when looking into the beam) travelling in the $-\hat{\mathbf{z}}$ direction.
D. Left circular polarized (electric field rotating counterclockwise when looking into the beam), travelling in the $-\hat{\mathbf{z}}$ direction.
E. Right circular polarized (electric field rotating clockwise when looking into the beam), travelling in the $+\hat{\mathbf{z}}$ direction.

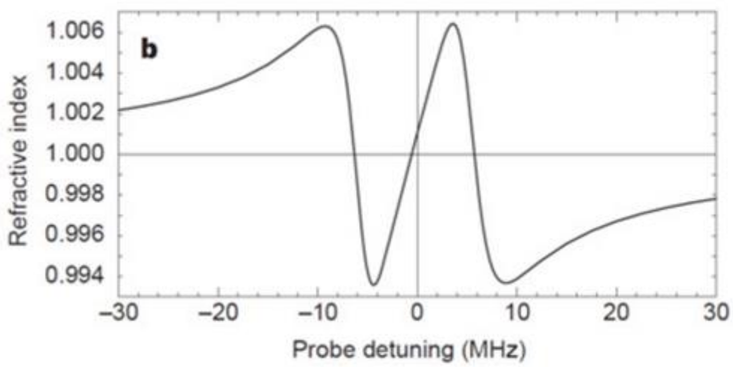
8) Which of the following is a correct unnormalized Jones vector for this light beam?

A. $4\begin{bmatrix}1\\1\end{bmatrix}$ V/m **B.** $4\begin{bmatrix}1\\i\end{bmatrix}$ V/m **C.** $4\begin{bmatrix}1\\-i\end{bmatrix}$ V/m **D.** $4\sqrt{2}\begin{bmatrix}1\\1\end{bmatrix}$ V/m **E.** $4\begin{bmatrix}1\\1\end{bmatrix}$ V/m

PART II – answer the following two questions in your exam booklet

9 a) The group velocity of a wavepacket is given by $\upsilon_g = d\omega / dk$, calculated at the angular frequency of the peak harmonic component, ω_{pk}. Starting with the general dispersion relation between angular frequency and propagation constant in terms of the refractive index, $n(\omega)$, of a medium, derive Eq. (6.48) from the text and your formula sheet.

b) It is experimentally possible to arrange two laser beams to be transmitted through a material despite there being very large dispersion. The figure below shows the variation of the refractive index of a material with light frequency for a wavelength region centred around 589 nm. Find the approximate minimum group velocity of a narrowband laser pulse of around 589 nm in this material, and compare this to the typical speed of a car on a highway.



10 a) <u>Fully</u> describe the polarization state of light described by the Jones vector below, and find $\mathbf{V}_{\text{norm}}$:

$$\mathbf{V}_1 = \begin{bmatrix} 2 \\ 2+i \end{bmatrix}$$

b) The principal refractive indices of quartz at an air wavelength of 589 nm are $n_{\parallel} = 1.5534$ and $n_{\perp} = 1.5443$. Find the minimum thickness of a plate of quartz that can make a $\lambda/8-\text{plate}$. Draw a labelled sketch of an experimental arrangement showing the orientations of the incoming beam and the crystal's optic axis for which this thickness of plate act as the desired $\lambda/8-\text{plate}$.

c) One example of a Jones matrix that adds a phase (this corresponding to a delay) of ε_x horizontally and a phase of ε_y vertically is

$$\mathbf{M}_{\text{waveplate}} = \begin{bmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{bmatrix}.$$

Using this find the *normalized* Jones vector, and <u>fully</u> describe the state of polarization, of the beam that emerges when light with Jones vector $\mathbf{V}_1$ above passes through a $\lambda/8-\text{plate}$ with its slow axis oriented *horizontally*.