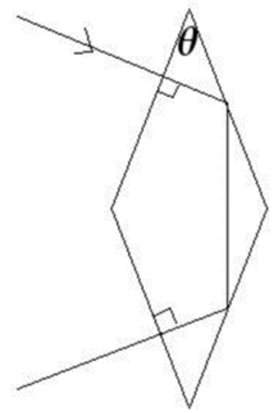
Trent University: PHYS 4240H *Modern Optics*: 2020-2021

Midterm: Tuesday November 4, 2020

Allowed: Open book, calculator, 1 hour 50 minutes
Answer all questions.
Each question is worth equal marks. Show your working; communicate your logic!

1) Consider as the optical element depicted below, composed of a rhomb (i.e. all sides the same length) and a path of light traveling through it, comprising two total internal reflections.

a) What value of the apex angle, θ, is required for the light to follow the path shown?



b) What is the *minimum* value of the refractive index of the rhomb such that light entering from air follows this desired path?

c) For this minimum refractive index what would be the wavelength *inside* the rhomb of incident light with a vacuum wavelength of $\lambda_0 = 500$ nm?

2) A plane monochromatic light wave traveling in vacuum can be described by the equation below representing the electric field, E_1, in V/m, with time *t* and distance *x* in seconds and meters, respectively:

$$E_1(x,t) = 870\sin\left(\omega t + 1.047\times10^7 x\right)$$

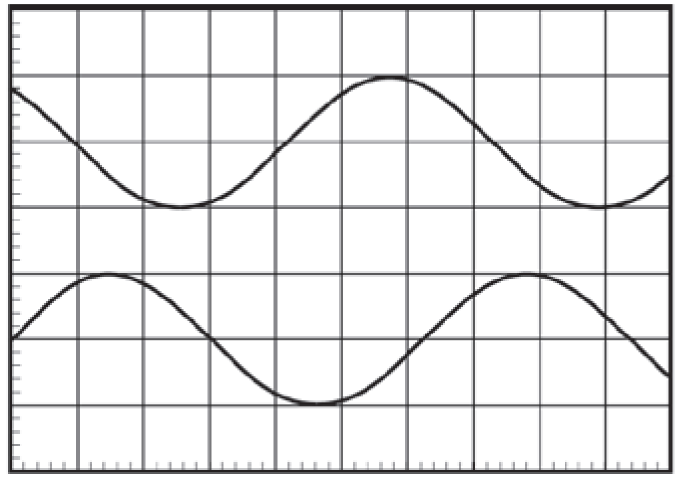
a) For this wave, what is: i) the wavelength λ_0, ii) the frequency ν, and iii) the propagation vector $\vec{k}$?

b) Suppose this wave hits a perfect reflector located at $x = 0$, with a phase delay due to reflection alone of π. What is the mathematical form of the **reflected wave**, described by $E_2(x,t)$?

c) What is the mathematical form of the resultant field, $E_R(x,t)$, in the region where these two waves overlap? Sketch this field, with appropriate labels, for *x* values from 0 to $5\lambda_0$.

d) [More difficult] If, instead, the reflector was placed at $x = x_0$, what then is the mathematical form of both the reflected wave and the resultant field?

3) Two harmonic oscillations are shown on the right.



a) Estimate a value for the phase difference, in degrees and in radians, that the bottom wave can be considered to be *ahead of* the top wave.

b) Similarly estimate a value for the phase difference, in degrees and in radians, that the bottom wave can be considered to be *behind* the top wave.

4) Figure 8-5 in the 3rd ed. of Pedrotti[3] depicts a Mach-Zehnder interferometer in which reflections occur at the *top* faces of beam splitter BS and mirror M2, and the *bottom* faces of mirror M1 and the second beam splitter (labelled M3, and called a "semitransparent mirror" in the text). Such selective reflections from an optical element such as a beam splitter can be accomplished by depositing a coating that results in *high-reflectivity* on one face and a coating resulting in *low reflectivity* on the other.

a) Derive and/or deduce (and carefully detail your reasoning and any sources you use) the general expression for the *minimum* thickness of a single layer of refractive index n_2 placed on a beam splitter like BS or M3 (with assumed refractive index $n_s > n_2$) that provides *maximum* reflection of an incident beam with vacuum wavelength λ_0 at incident angle $45°$. What is this thickness if $n_2 = 1.38$ and $\lambda_0 = 600\text{ nm}$?

b) Similarly, provide with reasoning the general formula for the *minimum* thickness of a single layer of refractive index n_2 placed on a beam splitter like BS or M3 (with assumed refractive index $n_s > n_2$) that provides *minimum* reflection of an incident beam with vacuum wavelength λ_0 at incident angle $45°$? What is this thickness if $n_2 = 1.38$ and $\lambda_0 = 600\text{ nm}$?

c) Describe what is meant by the terms 'external reflection' and 'internal reflection'. Using these terms carefully explain in words what is meant by the two Stokes relations given in Eq. (7-42) and (7-43) in Pedrotti[3]. Choose some plausible *numerical* values for r, r', t and t' and assuming that these apply equally to *both sides* of a glass plate show with a diagram how the values you have chosen feature in the first three reflected and transmitted rays emerging from the plate when incident light with amplitude E_0 hits the plate at some finite angle to the normal. Include numerical values on your diagram.

5) The group velocity, v_{g}, of a light pulse inside a medium is given by $v_{\mathrm{g}} = d\omega / dk$, determined for the *peak constituent harmonic wave* comprising the pulse – usually at either its angular frequency ω_{p}, or at its vacuum wavelength $\lambda_{0,\mathrm{p}}$.

a) Starting with the general dispersion relation between the angular frequency and the propagation constant in a medium, namely $k = \dfrac{\omega n(\omega)}{c}$, show that for a light pulse inside this medium:

$$v_{\mathrm{g}} = \frac{c}{n(\omega_{\mathrm{p}}) + \omega_{\mathrm{p}} \left.\dfrac{dn(\omega)}{d\omega}\right)_{\omega_{\mathrm{p}}}}$$

b) The refractive index of a material is usually measured as a function of *vacuum wavelength*, λ_0, instead of angular frequency ω, and so it is more convenient to express v_{g} in a way that it may be easily determined from the *experimentally-determined* curve of $n(\lambda_0)$ instead of $n(\omega)$. Derive yourself, either from the equation above for v_g or directly from the dispersion relation $k = \dfrac{\omega n(\lambda_0)}{c}$, a different expression for v_{g} with all terms now determined at the *peak vacuum wavelength*, $\lambda_{0,\mathrm{p}}$.

c) Use this result from (b) to then find v_{g} for a light pulse with peak vacuum wavelength 400 nm inside a medium where the refractive index as a function of vacuum wavelength is given by

$$n(\lambda_0) = 1.40 + \frac{2.5 \times 10^{-14}}{\lambda_0^{\;2}}$$

d) Equation (5-40) and Example 5-3 together in the 3rd ed. of Pedrotti[3] arrive at and use a slightly different expression for v_{g} than that of part (b) above, namely:

$$v_{\mathrm{g}} = \frac{c}{n(\lambda_{0,\mathrm{p}})} \left(1 + \frac{\lambda_{0,\mathrm{p}}}{n} \left.\frac{dn(\lambda_0)}{d\lambda_0}\right)_{\lambda_{0,\mathrm{p}}} \right)$$

Use this result to find Pedrotti's value of v_{g} for the same light pulse and material described in part (c).

e) [More difficult] Explain mathematically why the two values of v_{g} from (c) and (d) are different, but also so similar, to each other. Identify the discrepancy between them.