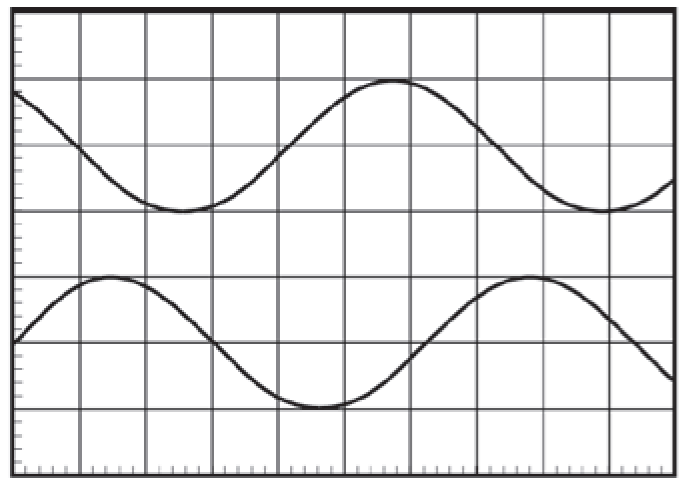
Trent University: PHYS 4240H – Modern Optics: 2012-2013

Midterm: Monday February 11, 2013

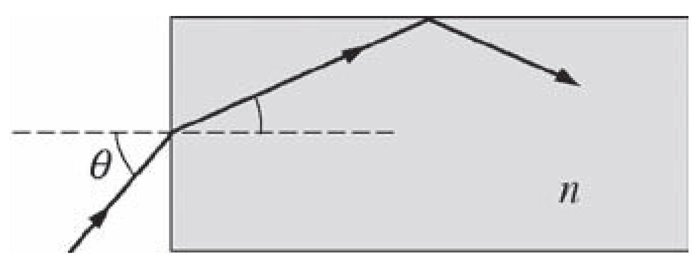
Allowed: Formula sheet (given), calculator, 1 hour 50 minutes
Answer <u>six</u> out of <u>seven</u> short-answer questions.
Each question is worth equal marks. Show your working!

1) Unpolarized light is incident on a pair of ideal linear polarizers whose transmission axes make an angle of 45° with each other. Find the transmitted light intensity through both polarizers as a percentage of the incident intensity.

2) Two sinusoids are shown on the right. Estimate the phase difference, in degrees and in radians, between them.



3) A model of an optical fiber is shown in the figure on the right. The optical fiber has an index of refraction, n, and is surrounded by free space. Show your working and determine which of the following angles of incidence, θ, will result in the light staying in the optical fiber.



a) $\theta > \sin^{-1}\left(\sqrt{n^2-1}\right)$

b) $\theta < \sin^{-1}\left(\sqrt{n^2-1}\right)$

c) $\theta > \sin^{-1}\left(\sqrt{n^2+1}\right)$

d) $\theta < \sin^{-1}\left(\sqrt{n^2+1}\right)$

e) $\sin^{-1}\left(\sqrt{n^2-1}\right) < \theta < \sin^{-1}\left(\sqrt{n^2+1}\right)$

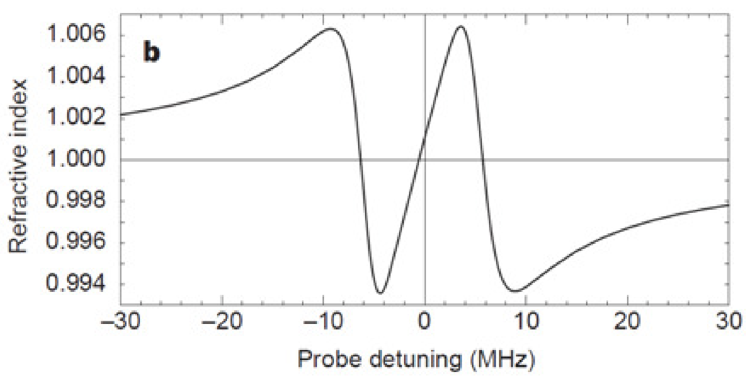
4) Starting with the equation for the irradiance of an electromagnetic field from the formula sheet, I, show that for a single monochromatic light beam travelling along the z-axis in free space with an electric field amplitude of E_0, then $I = \frac{1}{2} c\varepsilon_0 E_0^{\ 2}$

5) A Michelson interferometer forms fringes with cadmium red light (643.847 nm) and a linewidth of $\Delta\lambda = 0.0013\,\text{nm}$. i) Sketch the layout of a Michelson interferometer, and ii) estimate the visibility of the fringes when one mirror is moved 1 cm from the position of zero optical path difference.

6) The group velocity of a wavepacket, v_g, can be shown to be $v_g = d\omega / dk$ calculated at the value of the peak wavevector, k_p. Starting with the general relationship between refractive index, $n(\omega)$, and ω, show that:

$$v_g = \frac{c}{n(\omega_p) + \omega_p \left.\frac{dn(\omega)}{d\omega}\right)_{\omega_p}}$$

Modern optics experiments allow two laser beams to be transmitted with very large dispersion in a material. The figure below shows the variation of the refractive index of a material with frequency in a wavelength region centred at 589 nm. Estimate the group velocity of a narrowband laser pulse in this material and compare this to the typical speed of a car on a highway.



7) In the class notes we have seen that the dipole moment of an atom is proportional to the local electric field experienced by the atom, $\vec{E}_{local}$, while the bulk polarization of a material containing these atoms is proportional to the net macroscopic electric field inside the material, $\vec{E}$:

$$\vec{p} = \alpha \vec{E}_{local} \qquad\qquad \vec{P} = \varepsilon_0 \chi_e \vec{E}$$

Lorentz showed that for many materials $\vec{E}_{local} = \left(1 + \frac{\chi_e}{3}\right)\vec{E}$. From this, derive the Clausius-Mossotti (or Lorentz-Lorenz) formula relating the atomic polarizability, α, to the material's electrical susceptibility χ_e, for a material with an atomic number density of N:

$$\chi_e = \frac{N\alpha / \varepsilon_0}{1 - N\alpha / 3\varepsilon_0}$$