Trent University: PHYS 4240H – Modern Optics: 2010-2011

Midterm: Tuesday March 1, 2011

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes
Answer three out of four questions. Each question is worth equal marks. Show your working!

1. a) Michelson found that the cadmium red line (643.8 nm) was one of the most ideal monochromatic sources available, allowing fringes to be discerned up to a path difference of 30 cm in an interference experiment. Briefly explain why fringes from any light source are necessarily visible only up to some finite path difference and determine the wavelength spread, $\Delta\lambda$, the frequency spread, $\Delta\nu$, and the coherence time, τ_c, of this source.
b) Suppose we set up Young's double slit experiment using as a light source a small circular hole of diameter 0.1 mm placed in front of a sodium lamp (589.3 nm), with the two slits located 1 m away from the hole. Briefly explain why there is a maximum separation of the slits which will produce fringes on a screen placed after the slits, and estimate how far apart the slits can be before all fringes disappear.

2. Consider a (scalar) electric field in free space of the form: $E(z,t) = E_0 \; e^{-\alpha(z-ct)^2}$.
a) Sketch the spatial dependence of E at $t=0$, and again at some later value of t. What is the full width at half maximum (FWHM) of this curve?
b) Sketch the temporal dependence of E at $z=0$, and also at some greater value of z. What is the FWHM of this curve?
c) Given that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$, and by completing the square, show that:

$$E(z,0) = \frac{E_0}{2\sqrt{\pi\;\alpha}} \int_{-\infty}^{\infty} e^{-k^2/4\alpha} e^{ikz} dk$$

By comparing this expression with equation 1.8 on the formula sheet, find the FWHM of $|\tilde{g}(k)|^2$, the *power spectrum in* k. What would happen to both $E(z,0)$ and $|\tilde{g}(k)|^2$ as α decreases?

3. a) In a few sentences state the reasoning behind, and the approximations used to derive, the formula for $\tilde{\chi}_e(\omega)$ given on your formula sheet. By applying this formula to the case of dilute media, demonstrate that the real and imaginary parts of the *refractive index*, $\tilde{n}$, are given by:

$$n_R = 1 + \frac{Nq^2}{2m\varepsilon_0} \frac{\omega_0^{\;2} - \omega^2}{\left(\omega_0^{\;2} - \omega^2\right)^2 + \gamma^2\omega^2} \qquad \text{and} \qquad n_I = \frac{Nq^2}{2m\varepsilon_0} \frac{\gamma\omega}{\left(\omega_0^{\;2} - \omega^2\right)^2 + \gamma^2\omega^2}$$

b) Using the values: $\omega_0 = 1\times10^{16}$ s^{-1}; $\gamma = 10^{14}$ s^{-1}; and $N = 1\times10^{25}$ m^{-3}, calculate n_R for three closely-spaced values of ω_1, ω_2 and ω_3 given by $\omega_1 = 9.5\times10^{15}, \omega_2 = 9.6\times10^{15}$ s^{-1} and $\omega_3 = 9.7\times10^{15}$ s^{-1}. What are the phase velocities of light at each of these three frequencies? Estimate the *group velocity* of a pulse of light encompassing a number of frequencies between ω_1 and ω_3, and centred at ω_2.

4. a) From equations 2.11 and 2.14 on your formula sheet show that the *transmission coefficient* for light incident at $90°$ on a boundary between two non-magnetic dielectrics (so $H = B / \mu_0$), coming from the material with refractive index n_1 and towards the material with refractive index n_2 is:

$$t = \frac{2n_1}{n_1 + n_2} \ ,$$

where we have assumed that to a good approximation the refractive indices of the materials are real. Similarly derive the *reflection coefficient*, r .

b) Suppose a visible laser beam is incident on a thick glass plate with refractive index $n = 1.46$ at near-normal incidence. Find the irradiances of the three most-intense beams to leave the plate on each side of the plate, assuming unit incidence irradiance.