Physics 424H – Modern Optics
Midterm: Thursday 26th February.
Allowed: 1 hour, 30 mins. Calculator, formula sheet (given).
Answer all questions. Each question is worth equal marks. Show your working!

1. A wave train is represented by:

$$\tilde{f}(t) = 0 \qquad t < 0$$
$$\tilde{f}(t) = a\, i \exp\left(-\frac{t}{\tau}\right)\exp(i\omega_0 t) \qquad t \geq 0$$

with a and τ real, and $\tau >> 2\pi/\omega$

a) Sketch the form of the physical wave, $f(t)$
b) Using the same convention for the Fourier transform as shown on the formula sheet, find the Fourier transform of $\tilde{f}(t)$, i.e. $\tilde{g}(\omega)$.
c) Find the *power spectrum*, $\left|\tilde{g}(\omega)^2\right|$

2. The refractive index for electromagnetic waves propagating in an ionized gas is given by

$$n^2 = 1 - \omega'^2/\omega^2$$

Where ω', the *plasma frequency*, is determined by the density of the gas. Show that the product of the group and phase velocities is c^2.

3. a) Derive the reflection coefficient, r, and the transmission coefficient, t, for light traveling at normal incidence from one non-magnetic dielectric material of refractive index, n_1, to another of n_2.
b) A slab of GaAs crystal, used in a diode laser, has a refractive index of $n = 3.6$. What fraction of the intensity of the radiation generated within the slab and incident normally on one face is reflected?

4. A 1 kW laser beam has a cross-sectional diameter of 5 mm. Calculate the irradiance and the amplitudes of the electric and magnetic fields.

5. A pair of crossed polarizers, with axes at angles $\theta = 0°$ and $90°$ is placed in a beam of unpolarized light with irradiance I_0 so that light emerges from the first with $I_1 = \frac{1}{2} I_0$ and from the second with $I_2 = 0$. A third polarizer is placed between the two at angle $\theta = 45°$.
a) What then is I_2?
b) If the third polarizer instead rotates at ω, such that $\theta = 0$ at $t = 0$, show that:

$$I_2 = \frac{I_0}{16}(1 - \cos 4\omega t)$$

6. The reflectance, *R*, of the plates used in a Fabry-Perot interferometer is 60%. Find the visibility of the fringes formed from highly coherent light.