Trent: PHYS 4240H – Modern Optics: 2024-2025

# Final Exam: Tuesday Dec 17, 2024 Name:__________________

**Allowed: Formula sheet (given), calculator, 3 hours**

PART 1 – answer questions **1-10** in your exam booklet and **then** use your scratch card:
One scratch = **100%**; two scratches = **50%**; three scratches = **25%** (part 1 total is 60%)
PART 2 – answer any **two** questions in your exam booklet (each of these questions is worth 20%)

**Qu's 1-2**) Two monochromatic light beams, *beam 1* and *beam 2*, have electric fields of the following form:

Beam 1: $\vec{\mathbf{E}}_1(\vec{\mathbf{r}},t) = E_0\cos(kz-\omega t)\hat{\mathbf{y}}$

Beam 2: $\vec{\mathbf{E}}_2(\vec{\mathbf{r}},t) = E_0\cos(kz-\omega t)\hat{\mathbf{x}} + E_0\cos(kz-\omega t+\pi/2)\hat{\mathbf{y}}$

**1)** Which of the following statements best describes the polarizations of these two beams?

**A.** Beam 1 is linearly polarized along the *z*-axis, and beam 2 is left circular polarized light (electric field rotates counterclockwise when looking into the beam).
**B.** Beam 1 is linearly polarized along the *y*-axis, and beam 2 is right circular polarized light (electric field rotates clockwise when looking into the beam).
**C.** Beam 1 is linearly polarized along the *y*-axis, and beam 2 is left circular polarized light (electric field rotates counterclockwise when looking into the beam).
**D.** Beam 1 is linearly polarized along the *z*-axis, and beam 2 is left elliptical polarized light (electric field rotates counterclockwise when looking into the beam).
**E.** Beam 1 is linearly polarized along the *y*-axis, and beam 2 is right elliptical polarized light (electric field rotates clockwise when looking into the beam).

**2)** Which of the following are the intensities of these two beams?

**A.** $I_1 = \frac{1}{2}\varepsilon_0 c E_0^{\,2}$, $I_2 = \varepsilon_0 c E_0^{\,2}$

**B.** $I_1 = \frac{1}{2}\varepsilon_0 c E_0^{\,2}$, $I_2 = \frac{1}{2}\varepsilon_0 c E_0^{\,2}$

**C.** $I_1 = \varepsilon_0 c E_0^{\,2}$, $I_2 = \frac{1}{2}\varepsilon_0 c E_0^{\,2}$

**D.** $I_1 = \varepsilon_0 c E_0^{\,2}$, $I_2 = \varepsilon_0 c E_0^{\,2}$

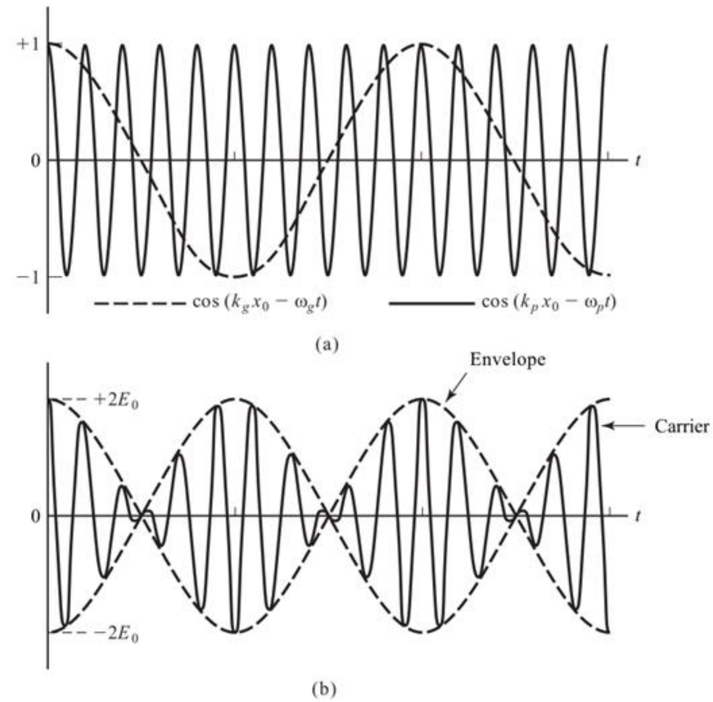
**E.** $I_1 = \varepsilon_0 c E_0^{\,2}$, $I_2 = 2\varepsilon_0 c E_0^{\,2}$

**Qu's 3-5)** Figures 6.7(a) and (b) from the course textbook are shown on the right. Figure 6.7(a) shows the two cosine factors of a resultant beat electric field of light at some location $x = x_0$:

$$E_R(x_0,t) = 2E_0 \cos(k_\mathrm{p} x_0 - \omega_\mathrm{p} t)\cos(k_\mathrm{g} x_0 - \omega_\mathrm{g} t).$$

Figure 6.7(b) shows (solid line) $E_R(x_0,t)$.

We shall assume that both plots cover the time duration from $t$ = 0 fs to $t$ = 30 fs.




**3)** Using Figure 6.7(a), which of the following are $\omega_\mathrm{p}$ and $\omega_\mathrm{g}$?

**A.** $\omega_\mathrm{p} = 500$ THz, $\omega_\mathrm{g} = 50$ THz

**B.** $\omega_\mathrm{p} = 10^{12}\pi$ s$^{-1}$, $\omega_\mathrm{g} = 10^{11}\pi$ s$^{-1}$

**C.** $\omega_\mathrm{p} = 4\times10^{12}\pi$ s$^{-1}$, $\omega_\mathrm{g} = 4\times10^{11}\pi$ s$^{-1}$

**D.** $\omega_\mathrm{p} = 10^{15}\pi$ s$^{-1}$, $\omega_\mathrm{g} = 10^{14}\pi$ s$^{-1}$

**E.** $\omega_\mathrm{p} = 2\times10^{15}\pi$ s$^{-1}$, $\omega_\mathrm{g} = 2\times10^{14}\pi$ s$^{-1}$

**4)** What therefore are the frequencies $\nu_1$ and $\nu_2$ of the two incoming waves of form $\cos(kx - 2\pi\nu t)$ that superpose to create the resultant electric field, $E_R(x,t)$?

**A.** $\nu_1 = 600$ THz, $\nu_2 = 400$ THz

**B.** $\nu_1 = 550$ THz, $\nu_2 = 450$ THz

**C.** $\nu_1 = 500$ THz, $\nu_2 = 50$ THz

**D.** $\nu_1 = 275$ THz, $\nu_2 = 225$ THz

**E.** $\nu_1 = 55$ THz, $\nu_2 = 45$ THz

**5)** What beat frequency would be recorded from measuring the resultant field with a photodetector?

**A.** $\nu_\mathrm{b} = 200$ THz **B.** $\nu_\mathrm{b} = 50$ THz **C.** $\nu_\mathrm{b} = 10$ THz **D.** $\nu_\mathrm{b} = 100$ THz **E.** $\nu_\mathrm{b} = 450$ THz

**Qu's 6-8)** The $S_3$ component of a Stokes vector can be found from the difference in intensity transmitted by two devices. We can therefore write $S_3 \equiv I_{\text{device 1}} - I_{\text{device 2}}$.

**6)** Which of the following best describes 'device 1' and 'device 2', which are used to provide $S_3$?

**A.** Device 1 passes without loss right-circular polarized light, and device 2 passes without loss left-circular polarized light.
**B.** Device 1 passes without loss horizontal linearly polarized light, and device 2 passes without loss vertical linearly polarized light.
**C.** Device 1 passes without loss right-circular polarized light, and device 2 passes without loss vertical linearly polarized light.
**D.** Device 1 passes without loss linearly polarized light at $+45°$, and device 2 passes without loss linearly polarized light at $-45°$.
**E.** Device 1 passes without loss left-circular polarized light, and device 2 passes without loss linearly polarized light at $-45°$.

**7)** By operating on the Jones vector of the light that we expect device 1 to pass without loss (so with no loss of intensity), which of the following Jones matrix expressions best represents device 1?

**A.** $\mathbf{M}_{\text{device 1}} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$

**B.** $\mathbf{M}_{\text{device 1}} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

**C.** $\mathbf{M}_{\text{device 1}} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

**D.** $\mathbf{M}_{\text{device 1}} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

**E.** $\mathbf{M}_{\text{device 1}} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

**8)** The *Mueller matrix* for device 1 is given by:

$$\mathfrak{M}_{\text{device 1}} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

What would be the transmittance ($T = I_{\text{out}} / I_{\text{in}}$) of device 1 when illuminated by light with Stokes vector $[1,0,0,1]$?

**A.** 28% **B.** 40% **C.** 64% **D.** 88% **E.** 100%

**Qu's 9-10)** A set of ideal linear polarizers are placed in succession and illuminated by unpolarized light. Suppose the intensity of the incoming light is $I_0$.

**9)** What is the intensity of the outgoing light if three linear polarizers are employed in succession, at angles $0°$, $45°$ and $90°$ to the horizontal?

**A.** $0.40 I_0$ **B.** $0.25 I_0$ **C.** $0.33 I_0$ **D.** $0.125 I_0$ **E.** $0.15 I_0$

**10)** What is the intensity of the outgoing light if seven linear polarizers are employed in succession, at angles $0°$, $15°$, $30°$, $45°$, $60°$, $75°$ and $90°$ to the horizontal?

**A.** $0.40 I_0$ **B.** $0.25 I_0$ **C.** $0.33 I_0$ **D.** $0.125 I_0$ **E.** $0.15 I_0$

**PART II – answer two of the following questions in your exam booklet**

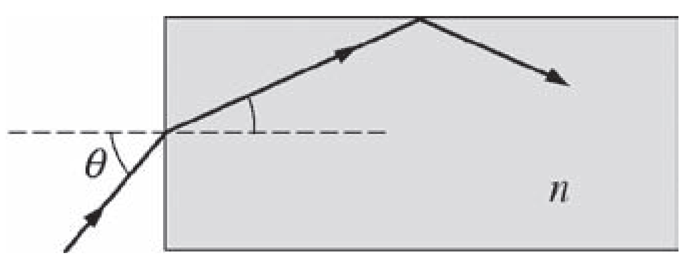
**11.** Consider a 1 km parabolic GRIN fiber with a peak core refractive index of 1.46 and a cladding refractive index of 1.44.

**a)** Briefly explain what is meant by a *parabolic GRIN fiber*, and why it serves its desired purpose.

**b)** Calculate the delay due to modal dispersion in this fiber.

**c)** Calculate this delay for a step-index fiber with $n_1 = 1.46$ and $n_2 = 1.44$. By what factor is this improved with the GRIN fiber?

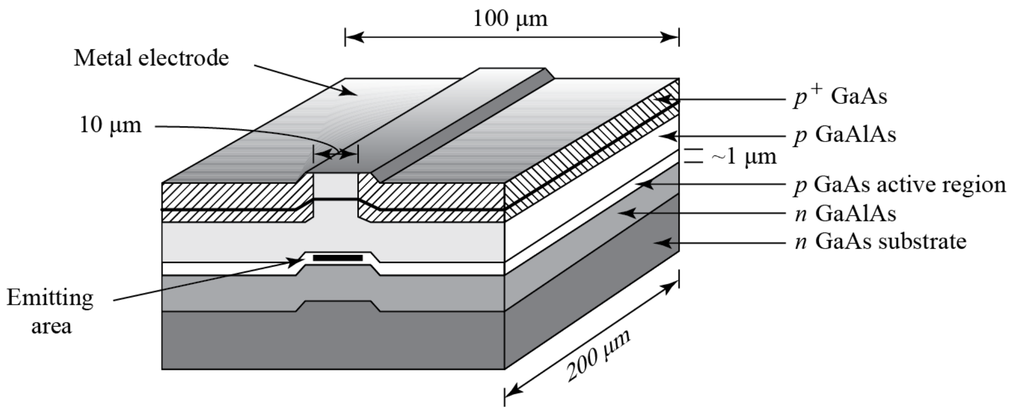
**12.** A simple model of a cylindrical optical fiber is shown on the right. This fiber has a refractive index *n*, and is entirely surrounded by air.




**a)** Find an expression in terms of only *n* for $\sin(\theta_{max})$, where $\theta_{max}$ is the largest angle of incidence that results in light staying fully within the fiber.

**b)** Derive the formula for the approximate number of skips that light entering with an arbitrary $\theta < \theta_{max}$ will make in this fiber of length *L* and diameter *d*, assuming this ray remains in a plane containing the axis of the fibre (i.e. it is a *meridional ray*). Find this value for $\theta = \theta_{max}/2$ for $n = 1.4$, $L = 2$ m and $d = 100$ μm.

**13.** Consider a laser diode of the type shown below:




**a)** Given that the band gap of intrinsic GaAs is 1.424 eV, what wavelength would be emitted from intrinsic GaAs?

**b)** Briefly describe what is meant by "p GaAs" in this diagram.

**c)** Describe the shape of the emerging beam, find the far-field half-angle divergence angles(s) of this beam in degrees, and find the approximate total dimensions of this beam 3 m from the laser.

**d)** The refractive index of GaAs at the wavelength emitted by this laser is approximately 3.67. Approximately how many half-wavelengths of laser light would fit within the cavity formed by the end faces of this laser?

**14.** An erbium fiber laser has a refractive index of 1.46 and an output wavelength of 1550 nm. To a reasonable approximation it can be viewed as a three-level laser system, comprising levels $|1\rangle$, $|2\rangle$ and $|3\rangle$, with upper laser level $|2\rangle$ decaying almost entirely to lower laser level $|1\rangle$ with a lifetime of 10 ms. Consider the laser transition to be inhomogeneously broadened with a Gaussian lineshape and a linewidth of 5 THz.

**a)** Briefly describe with the help of a diagram what is meant by *inhomogeneously broadened*.

**b)** Determine an expression for the peak stimulated emission cross-section of the lasing transition, $\sigma_{21}(\nu_{12})$, in terms of its linewidth $\Delta\nu$ and the spontaneous decay rate of the transition, $A_{21}$.

**c)** What is the value of $\sigma_{21}(\nu_{12})$? [To allow for the effect of the refractive index of the fiber, replace $c$ in this expression by $c/n$].