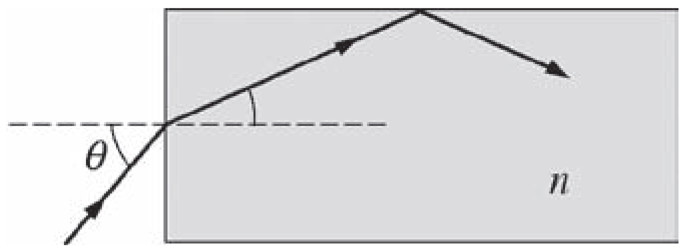
Final Exam: Saturday December 19, 2020

Allowed: Open book (so give references for all material used), calculator, 3 hours
Answer <u>all</u> questions.
Each question is worth equal marks. Show your working; communicate your thinking!

1) A model of an optical fiber is shown on the right. This fiber has index of refraction, *n*, is surrounded by air (treat $n_{\text{air}} \approx 1$), and assumed to be cylindrical along the horizontal axis.



a) Determine in its simplest mathematical form the expression for the <u>largest</u> angle of incidence, θ_{max}, which will result in light staying fully within the fiber.

b) Determine the approximate number of bounces that light entering with an arbitrary $\theta < \theta_{\text{max}}$ will make in such a long fiber of length *L* and diameter *d*, assuming this ray lies within a vertical plane containing also the axis of the fibre (i.e. it is a *meridional ray*).

c) The lowest-loss beam that can travel down the fiber satisfies the following approximate *dispersion relation* between its propagation constant along the axis, *k*, and its angular frequency, ω:

$$k = \frac{\omega n^2}{c}\left(1 - \frac{2\pi^2 c^2}{d^2 n^2 \omega^2}\right)$$

Given this expression, and neglecting any dependence of *n* on ω, determine the *group velocity* of a pulse of light with angular frequency centred at ω_p that follows this path.

2) A wave packet is represented by the complex function:

$$\tilde{f}(t) = 0 \qquad\qquad t < 0$$

$$\tilde{f}(t) = a\, i \exp\left(\frac{-t}{\tau}\right)\exp(-i\omega_0 t) \qquad t \geq 0$$

with a and τ real, and $\tau >> 2\pi / \omega_0$

a) Find the mathematical expression for, and sketch the form of, the <u>physical</u> wave, $f(t)$.
b) Using the same convention for the Fourier transform as used by Chapter 9, in Pedrotti, 3rd edition, find the Fourier transform of $\tilde{f}(t)$, i.e. $\tilde{g}(\omega)$.

c) Determine and sketch on the same graph the *power spectrum*, $\left|\tilde{g}(\omega)\right|^2$, for two different (one large, one small) values of τ, and clearly identify each trace.

3) a) Consider the energy level diagram for a laser given in Fig 6-15a in Pedrotti 3rd edition, and adopt the variables R_1, R_2 and R_4 for the rates of the (we assume) *unidirectional* steps labelled 1,2, and 4 respectively. Determine and write down the four coupled differential (i.e. rate) equations for the populations N_0, N_1, N_2 and N_3 in each level in terms of: these three variables; the intensity of the laser light in the cavity, I ; the value of the transition lineshape at the laser frequency, $g(\nu')$; and the relevant Einstein coefficients. (Note that it may help to look at, and to extend the set of, equations numbered 26-12 and 26-13).

b) The behaviour of a Nd:YAG laser can be represented by such a four-level energy diagram. Given this, copy down the figure and provide energies, in eV, of each of the relevant levels above the ground state. What approximate wavelength would be required to optically pump this laser?

4) a) Looking into a Michelson interferometer that uses a light source with $\lambda = 600\text{ nm}$, one can see a dark central disk surrounded by concentric bright and dark rings. One arm of the device is 1.5 cm longer than the other. Determine the order of this central disk.

b) Figure 7-1 in Pedrotti 3rd edition shows wavefronts corresponding to two monochromatic, same-frequency plane waves that intersect at an angle that we shall call θ. Treating the wavefronts as peaks of the respective waves at some instant of time, and assuming that the electric fields of the two waves are parallel to each other, determine and carefully describe the shape of the *interference fringes* that occur within the region of beam overlap. Find in terms of λ and θ,the <u>spacing</u> of the fringes (i.e. the distance between *adjacent bright*, or *adjacent dark*, fringes) along the line perpendicular to the bisector of the two incoming rays. (You may find it helpful to make several sketches as you work through this).