Trent University: PHYS 4240H – Modern Optics: 2012-2013

Final Exam: Saturday 20th April 2013

Allowed: **Time – 3 hours, formula sheet (given) and calculator**

Answer **all** questions from **Section A** (each has equal marks), and **two** out of three questions from **Section B**. Both sections are worth equal marks.

Section A

A1. Calculate whether the spontaneous emission rate or the stimulated emission rate is the dominant process for an atom inside an oven with a transition frequency of 10^{10} Hz , illuminated by blackbody light at 300 K. What part of the electromagnetic spectrum corresponds to 10^{10} Hz ?

A2. The *dispersion relation* that relates angular frequency, ω, to the wavevector, k, for electromagnetic waves in a plasma composed of free ions and electrons each with number density N is given by:

$$\omega^2(k) = \omega'^2 + c^2k^2 ,$$

where the *plasma angular frequency*, ω', is defined as (m and e being the electron mass and charge):

$$\omega'^2 = \frac{Ne^2}{m\varepsilon_0} .$$

a) Find the plasma angular frequency of the Earth's ionosphere, where $N \approx 10^5$ cm^{-3}. What part of the electromagnetic spectrum does this angular frequency correspond to?

b) Show that the refractive index for electromagnetic waves propagating in such a plasma is given by

$$n^2 = 1 - \frac{\omega'^2}{\omega^2}$$

c) Show that the product of the *phase* and *group* velocities of a wavepacket travelling in this plasma is given by c^2.

A3. A wave packet is represented by the complex scalar field:

$$\tilde{f}(t) = 0 \qquad\qquad t < 0$$

$$\tilde{f}(t) = a\, i \exp\left(-\frac{t}{\tau}\right)\exp(i\omega_0 t) \qquad\qquad t \geq 0$$

with a and τ real, and $\tau >> 2\pi/\omega_0$

a) Sketch the form of the physical wave, $f(t)$

b) Write down the convention you wish to use for the Fourier transform of $\tilde{f}(t)$, and then find its Fourier transform, i.e. $\tilde{g}(\omega)$.

A4. Calculate the Doppler linewidth, $\Delta\nu$ in MHz, of the transition of ^{20}Ne in a HeNe laser operating on the 632.8 nm transition, assuming the gas temperature is 400 K.

A5. a) Briefly describe what is meant by the *complex degree of coherence*, $\tilde{\gamma}_{12}(\tau)$ of a light field, and explain the terms and operations involved in the equation that defines it.

b) Suppose that the complex degree of coherence of a light field with central angular frequency ω is given by:

$$\tilde{\gamma}_{12}(\tau) = \exp\left(-4\ln 2\,\tau^2/\tau_c^{\ 2}\right)\exp(i\omega\tau)$$

where τ_c is a constant. By finding expressions for both the maximum and minimum intensities of a fringe pattern resulting from two equally-illuminated slits which are separated by distance b in a Young's interference experiment, show that $V(\theta)$, the visibility of observed fringes at an angle of θ to the perpendicular bisector between the slits is given by:

$$V(\theta) = \exp\left(-\frac{4\ln 2\, b^2\,\theta^2}{c^2\,\tau_c^{\ 2}}\right).$$

Section B

B1. A plane monochromatic electromagnetic wave traveling in *vacuo* and incident on a material has the following electric field and magnetic fields:

$$\mathbf{E_I}(\vec{r},t) = E_{0I}\hat{\mathbf{x}}\cos(kz-\omega t)$$
$$\mathbf{B_I}(\vec{r},t) = B_{0I}\hat{\mathbf{y}}\cos(kz-\omega t) \qquad \text{(Eq. 1)}$$

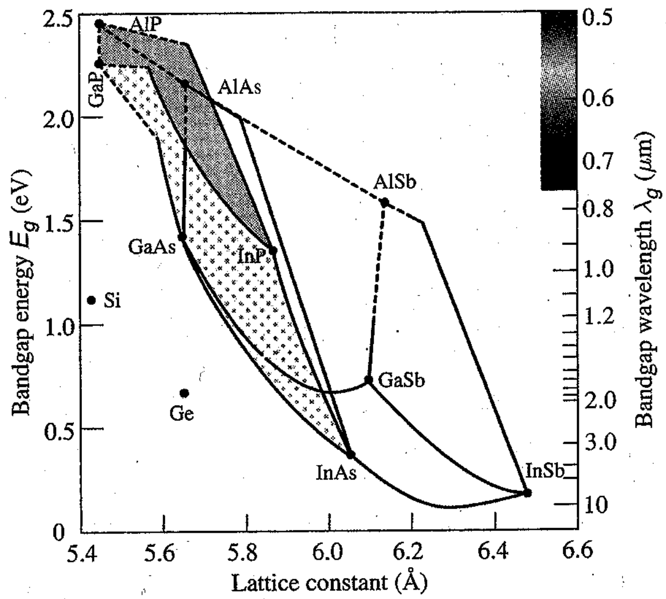
a) Depict this wave, paying careful attention to the relationship between the fields and the relevant axes. What is the boundary condition for the electric field parallel to the interface on either side of the interface between two media?

b) Find the relationship between E_{0I} and B_{0I} for such a single traveling wave, and also find the irradiance of this wave in terms of E_{0I}.

c) Suppose this wave hits the surface of a perfect reflector (reflection coefficient $r=-1$) at normal incidence placed at z = 0. Write down equations similar to Eq. 1 for the electric and magnetic fields of the *reflected* wave in terms of E_{0I}, k, ω, and ensure that each field has the correct phase (i.e. pure cosine or pure sine or something in between) and direction. Depict the incident and reflected electric and magnetic fields.

d) Describe the resultant electric and magnetic fields for z < 0 both qualitatively *and* algebraically.

B2. a) In approximately four sentences, explain the information that is given in the diagram below:



b) The table below lists the properties of selected transitions in particular laser gain media found in different lasers:

Laser Medium	Transition Wavelength[a] λ_o (nm)	Transition Cross Section σ_0 (cm^2)	Spontaneous Lifetime t_{sp}	Transition Linewidth[b] $\Delta\nu$		Refractive Index n
C^{5+}	18.2	5×10^{-16}	12 ps	1 THz	I	≈ 1
ArF Excimer	193	3×10^{-16}	10 ns	10 THz	I	≈ 1
Ar^+	515	3×10^{-12}	10 ns	3.5 GHz	I	≈ 1
Rhodamine-6G dye	560–640	2×10^{-16}	5 ns	40 THz	H/I	1.40
He–Ne	633	3×10^{-13}	150 ns	1.5 GHz	I	≈ 1
Cr^{3+}:Al_2O_3	694	2×10^{-20}	3 ms	330 GHz	H	1.76
Cr^{3+}:$BeAl_2O_4$	700–820	1×10^{-20}	260 μs	25 THz	H	1.74
Ti^{3+}:Al_2O_3	700–1050	3×10^{-19}	3.9 μs	100 THz	H	1.76
Yb^{3+}:YAG	1030	2×10^{-20}	1 ms	1 THz	H	1.82
Nd^{3+}:Glass (phosphate)	1053	4×10^{-20}	370 μs	7 THz	I	1.50
Nd^{3+}:YAG	1064	3×10^{-19}	230 μs	150 GHz	H	1.82
Nd^{3+}:YVO_4	1064	8×10^{-19}	100 μs	210 GHz	H	2.0
InGaAsP[c]	1300–1600	2×10^{-16}	2.5 ns	10 THz	H	3.54
Er^{3+}:Silica fiber	1550	6×10^{-21}	10 ms	5 THz	H/I	1.46
CO_2	10 600	3×10^{-18}	3 s	60 MHz	I	≈ 1

From: *Fundamentals of Photonics*, Saleh & Teich

i) For **each** of the lasers listed, state the type of each laser (e.g. "plasma laser", "gas laser", "solid state laser", "semiconductor laser", "liquid laser"). What do the "I" and "H" signify?

ii) For a laser of type "I", determine a general expression for the peak cross-section of the transition, $\sigma_0 \equiv \sigma_{21}(\nu_0)$, in terms of its linewidth $\Delta\nu$ and the spontaneous decay rate of the transition, A_{21}. Explain with the help of a diagram why A_{21} does not always equal the total spontaneous decay rate of the state 2, A_2. For the HeNe laser first find A_2, and then, given that the spontaneous decay rate of the lasing transition is 3.03×10^6 s^{-1}, verify the value of σ_0 given in the table.

B3) Consider a conventional laser composed of a back mirror with reflectivity of 99% and an output coupler with reflectivity of 98% and transmissivity of 1%. The gain medium is a 20 cm Nd:YAG crystal with a saturation intensity of 2300 W/cm^2 that is pumped resulting in a small-signal gain coefficient of 0.05 cm^{-1}. With the help of a diagram that clearly shows the counterpropagating beams in this cavity, and assuming steady-state operation:
i) Find the *threshold gain coefficient*.
ii) Find the irradiance of the light that exits from the output coupler.
iii) What is the total irradiance at the end of the gain medium nearest to the output coupler?
iv) What is the corresponding gain coefficient at this location, and how does it compare with the threshold gain coefficient?