Trent University: PHYS 4240H – Modern Optics: 2010-2011

Final Exam: Monday 18th April 2011

Allowed: **Time – 3 hours, formula sheet (given) and calculator**

Answer **all** questions from **Section A**, and **two** out of three questions from **Section B**.
Both sections are worth equal marks.

Section A

A1 Determine the linewidth in both angstroms and hertz for light from a HeNe laser at 6328 Å whose coherence length is 10 km.

A2. A wave packet is represented by the complex scalar field:

$$\tilde{f}(t)=0 \qquad\qquad t<0$$

$$\tilde{f}(t)=a\,i\exp\left(-\frac{t}{\tau}\right)\exp(i\omega_0 t) \qquad\qquad t\geq 0$$

with a and τ real, and $\tau >> 2\pi/\omega_0$

a) Sketch the form of the physical wave, $f(t)$

b) Write down the convention you wish to use for the Fourier transform of $\tilde{f}(t)$, and then find its Fourier transform, i.e. $\tilde{g}(\omega)$.

c) Find the *power spectrum*, $\left|\tilde{g}(\omega)^2\right|$

A3. An air-spaced Fabry-Perot interferometer has spacing 1 cm and reflection coefficient of the mirrors of 0.95. Determine the finesse of this interferometer, $\Im$, and, for a wavelength of 500 nm calculate its minimum resolvable wavelength, $\Delta\lambda$, and its resolving power, $\lambda/\Delta\lambda$.

A4. The complex refractive index of the semiconductor cadmium telluride is measured to be $\tilde{n}=3.01+0.38\,\mathrm{i}$ for incident light with a wavelength of 500 nm. By considering a monochromatic plane wave, $\tilde{E}(z,t)$, travelling within CdTe along the z-axis, calculate the phase velocity, the wavelength and the distance travelled before the irradiance has decreased to 1/e of the original value.

A5. Calculate the linewidth, $\Delta\nu$ in MHz, of the transition of ^{20}Ne in a HeNe laser operating on the 632.8 nm transition, assuming the gas temperature is 400 K.

Section B

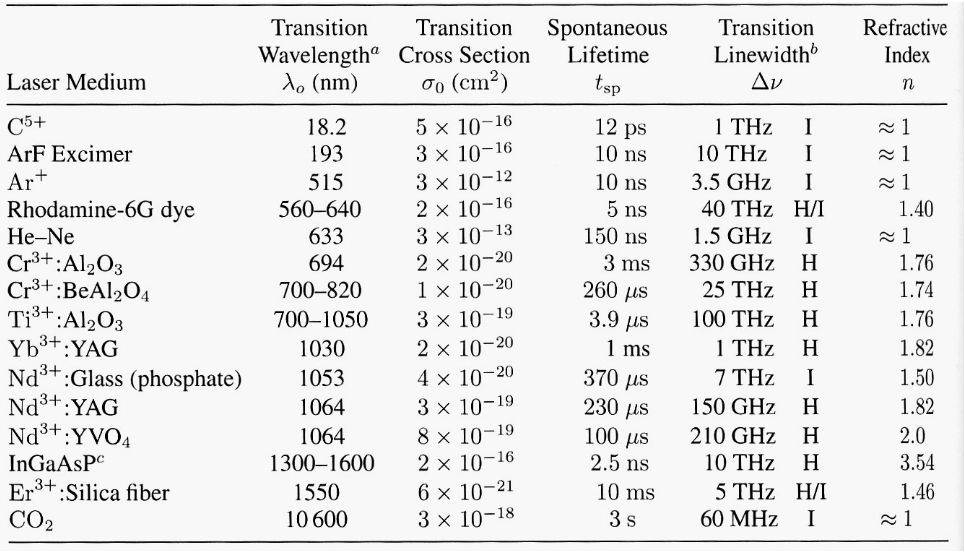
B1. a) In a few sentences state the reasoning behind, and the approximations used to derive, the formula for $\tilde{\chi}_e(\omega)$ given on your formula sheet. By applying this formula to the case of dilute media, demonstrate that the real and imaginary parts of the *refractive index*, $\tilde{n}$, are given by:

$$n_R=1+\frac{Nq^2}{2m\varepsilon_0}\frac{\omega_0^{\,2}-\omega^2}{\left(\omega_0^{\,2}-\omega^2\right)^2+\gamma^2\omega^2} \qquad \text{and} \qquad n_I=\frac{Nq^2}{2m\varepsilon_0}\frac{\gamma\omega}{\left(\omega_0^{\,2}-\omega^2\right)^2+\gamma^2\omega^2}$$

Make a rough sketch of both $n_R(\omega)$ and $n_I(\omega)$.

b) Using the values: $\omega_0 = 1\times10^{16}$ s^{-1}; $\gamma = 10^{14}$ s^{-1}; and $N = 1\times10^{25}$ m^{-3}, calculate n_R for three closely-spaced values of ω_1, ω_2 and ω_3 given by $\omega_1 = 9.5\times10^{15}$, $\omega_2 = 9.6\times10^{15}$ s^{-1} and $\omega_3 = 9.7\times10^{15}$ s^{-1}. What are the phase velocities of light at each of these three frequencies? Estimate the *group velocity* of a pulse of light encompassing a number of frequencies between ω_1 and ω_3, and centred at ω_2.

B2. The table below lists laser gain materials for a selection of different lasers:



Laser Medium	Transition Wavelength[a] λ_o (nm)	Transition Cross Section σ_0 (cm^2)	Spontaneous Lifetime t_{sp}	Transition Linewidth[b] $\Delta\nu$		Refractive Index n
C^{5+}	18.2	5×10^{-16}	12 ps	1 THz	I	≈ 1
ArF Excimer	193	3×10^{-16}	10 ns	10 THz	I	≈ 1
Ar^+	515	3×10^{-12}	10 ns	3.5 GHz	I	≈ 1
Rhodamine-6G dye	560–640	2×10^{-16}	5 ns	40 THz	H/I	1.40
He–Ne	633	3×10^{-13}	150 ns	1.5 GHz	I	≈ 1
Cr^{3+}:Al_2O_3	694	2×10^{-20}	3 ms	330 GHz	H	1.76
Cr^{3+}:$BeAl_2O_4$	700–820	1×10^{-20}	260 μs	25 THz	H	1.74
Ti^{3+}:Al_2O_3	700–1050	3×10^{-19}	3.9 μs	100 THz	H	1.76
Yb^{3+}:YAG	1030	2×10^{-20}	1 ms	1 THz	H	1.82
Nd^{3+}:Glass (phosphate)	1053	4×10^{-20}	370 μs	7 THz	I	1.50
Nd^{3+}:YAG	1064	3×10^{-19}	230 μs	150 GHz	H	1.82
Nd^{3+}:YVO_4	1064	8×10^{-19}	100 μs	210 GHz	H	2.0
InGaAsP[c]	1300–1600	2×10^{-16}	2.5 ns	10 THz	H	3.54
Er^{3+}:Silica fiber	1550	6×10^{-21}	10 ms	5 THz	H/I	1.46
CO_2	10 600	3×10^{-18}	3 s	60 MHz	I	≈ 1

From: *Fundamentals of Photonics*, Saleh & Teich

a) For **each** of the lasers listed, state the type of each laser (e.g. "plasma laser", "gas laser", "solid state laser", "semiconductor laser", "liquid laser").
b) For **one** example of **each of these 5 types** of laser from the table, briefly (in one or two sentences) describe why these have either an "I", "H" or "H/I "in the *Transition Linewidth* column.
c) For a typical laser of type "I", determine an expression for the peak cross-section of the transition, $\sigma_0 \equiv \sigma(\nu_0)$, in terms of its linewidth $\Delta\nu$ and the spontaneous decay rate of the transition, A_{21}. Explain with the help of a diagram why A_{21} does not always equal the total spontaneous decay rate of state 2, A_2. For the CO_2 laser first find A_2, and then, given that the spontaneous decay rate of the lasing transition is 4.35×10^{-3} s^{-1}, verify the value of σ_0 given in the table.

B3. Consider a unidirectional ring laser where light travels from mirror 1 (99% reflective) to mirror 2 (99% reflective), through a gain medium, and towards the output coupler (95% reflective, 4% transmission) before being either transmitted out of the laser or reflected back to mirror 1. The gain medium is a 10 cm long Nd:YAG crystal with a saturation irradiance of 2300 W/cm^2 and the crystal is pumped at a rate that results in a small-signal gain coefficient of 0.05 cm^{-1}. Assuming steady state operation:
a) Make a sketch of the ring laser, and find the threshold gain coefficient of this cavity.
b) Find the irradiance of the laser field that exits from the output coupler.
c) What is the irradiance at each end of the gain medium inside the cavity? What are the corresponding values of the gain coefficient, γ, and how do these compare with the threshold gain coefficient? What is the name given to this phenomenon, where the gain coefficient reduces from its small-signal value?