Physics 424H – Modern Optics

Final Exam: Tuesday 21st April 2009

Allowed: 3 hours, calculator, formula sheet (given).
Answer **all** questions from Section A, and **two** questions from Section B.
Both sections are worth equal marks.

Section A

A1 a) A beam of linearly polarized light with irradiance of 200 W/m^2 and its electric field vertical impinges perpendicularly on a linear polarizer with a transmission axis at $37°$ to the vertical. What is the irradiance of the transmitted beam?

b) A light wave has an electric field given by (all values are given in S.I. units):

$$\vec{\mathbf{E}}(z,t) = 870\,\hat{\mathbf{x}}\cos\left(9.94\times10^{6}\,z - 2.98\times10^{15}\,t\right) - 870\,\hat{\mathbf{y}}\sin\left(9.94\times10^{6}\,z - 2.98\times10^{15}\,t\right).$$

Determine the wavelength, the wave vector, the period, the velocity and the irradiance (in appropriate units). With the aid of a diagram, carefully describe the polarization of this wave.

A2. a) Briefly describe what is meant by the *complex degree of coherence*, $\tilde{\gamma}_{12}(\tau)$, and why it is a useful concept. What is meant by τ ?

b) Suppose that the degree of coherence from two equally-illuminated slits separated by distance *b* in a Young's interference experiment can be described by:

$$\tilde{\gamma}_{12}(\tau) = \exp\left(-4\ln 2\,\tau^2/\tau_c{}^2\right)\exp(i\omega\tau)$$

where τ_c is a constant. Find an expression for $V(\theta)$, the visibility of the fringes at an angle of θ to the perpendicular bisector between the slits.

A3. Determine **all** the constituent signal frequencies within the total irradiance field formed by two overlapping, collinear laser beams, each of equal irradiance, and with wavelengths 671.4260 nm and 671.4261 nm, respectively. Which of these frequencies are observable using modern detectors?

A4. A continuous and stable He-Ne laser at 632.8 nm is chopped using a spinning aperture into 1 μs pulses. Compute the resultant linewidth $\Delta\lambda$, bandwidth $\Delta\nu$ and coherence length, ℓ_c.

A5. Consider an atomic system modeled by a set of 10^{28} classical oscillators/m^3, each with resonant angular frequency of $\omega_0' = 10^{16}$ s^{-1} and damping constant of 10^{14} s^{-1}. Calculate the complex *dielectric constant* (also called complex *relative permittivity*) at $\omega = \omega_0'$, and $\omega = 0.9\omega_0'$.

A6. Calculate the linewidth, $\Delta\nu$ in MHz, of the absorption profile for the $^{12}C^{16}O_2$ isotope on the 10.6 μm transition, assuming the gas temperature is 373 K.

Section B

B1. a) Describe, with a diagram, the operation of a *Fabry-Perot interferometer*. If the distance between the two reflective surfaces is d and the material has refractive index n_2, while that outside the interferometer has corresponding index n_1, show that the optical path difference (opd) between two adjacent, parallel, transmitted rays is:

$$\text{opd} = 2n_2 d\cos\theta_t\,,$$

where θ_t is the angle of refraction of these rays inside the interferometer.

b) Give the condition on d, n_2, θ_t, and the incident vacuum wavelength, λ_0, for a bright fringe to be formed.

c) Suppose such an air-spaced interferometer has spacing $d \approx 5\,\text{cm}$ and reflection coefficient, $r = 0.97$. Sketch the shape of the transmitted irradiance that results from a single incident wavelength at a constant incident angle $\theta_i \approx 0$, as the distance d is scanned. What is the free-spectral range, $\Delta\nu_{fsr}$, of this interferometer, and what does this term mean in practice?

B2. Draw the energy level scheme for an optically-pumped *four-level* laser, with levels $|0\rangle$, $|1\rangle$, $|2\rangle$ and $|3\rangle$.

a) Including all *stimulated* absorption/decay rates ($\approx \sigma I/h\nu$), and *non-stimulated* decay rates ($\approx K_{ij}$), write down rate equations for each of N_0, N_1, N_2 and N_3; the numbers of atoms per unit volume in each level.

b) When these equations are solved for the steady state condition, *gain saturation* is evident. What is meant by this term and what equation reflects this phenomenon?

c) Suppose 20 cm of this laser material is housed within a cavity formed by two plane, parallel mirrors, with reflectivity of 0.99, and 0.98, respectively. What is the minimum small-signal gain required for lasing?

B3. The table below lists laser gain materials for a selection of different lasers:

Laser Medium	Transition Wavelength[a] λ_o (nm)	Transition Cross Section σ_0 (cm^2)	Spontaneous Lifetime t_{sp}	Transition Linewidth[b] $\Delta\nu$		Refractive Index n
C^{5+}	18.2	5×10^{-16}	12 ps	1 THz	I	≈ 1
ArF Excimer	193	3×10^{-16}	10 ns	10 THz	I	≈ 1
Ar^+	515	3×10^{-12}	10 ns	3.5 GHz	I	≈ 1
Rhodamine-6G dye	560–640	2×10^{-16}	5 ns	40 THz	H/I	1.40
He–Ne	633	3×10^{-13}	150 ns	1.5 GHz	I	≈ 1
Cr^{3+}:Al_2O_3	694	2×10^{-20}	3 ms	330 GHz	H	1.76
Cr^{3+}:$BeAl_2O_4$	700–820	1×10^{-20}	260 μs	25 THz	H	1.74
Ti^{3+}:Al_2O_3	700–1050	3×10^{-19}	3.9 μs	100 THz	H	1.76
Yb^{3+}:YAG	1030	2×10^{-20}	1 ms	1 THz	H	1.82
Nd^{3+}:Glass (phosphate)	1053	4×10^{-20}	370 μs	7 THz	I	1.50
Nd^{3+}:YAG	1064	3×10^{-19}	230 μs	150 GHz	H	1.82
Nd^{3+}:YVO_4	1064	8×10^{-19}	100 μs	210 GHz	H	2.0
InGaAsP[c]	1300–1600	2×10^{-16}	2.5 ns	10 THz	H	3.54
Er^{3+}:Silica fiber	1550	6×10^{-21}	10 ms	5 THz	H/I	1.46
CO_2	10 600	3×10^{-18}	3 s	60 MHz	I	≈ 1

From: *Fundamentals of Photonics*, Saleh & Teich

a) For **each** of the lasers listed, state the type of each laser (e.g. "plasma laser", "gas laser", "solid state laser", "semiconductor laser", "liquid laser").

b) For **one** example of **each of the 5 types** of laser from the table, briefly (in one or two sentences) describe why these have either an "I", "H" or "H/I "in the *Transition Linewidth* column.

c) What is the expected relationship between peak cross-section, $\sigma_0 \equiv \sigma(\nu_0)$, transition linewidth, $\Delta\nu$, and spontaneous lifetime, t_{sp}? Show that this relationship approximately holds for the Erbium fiber laser (assume the linewidth to be dominated by "I" broadening, and do not neglect the refractive index of the material).