Physics 424H – Modern Optics

Final Exam: Saturday 14th April 2007

Allowed: 3 hours, calculator, up to 2 sides of 8 ½”× 11” paper (equations only, no text, no figures).
Answer **all** questions from Section A, and **two** questions from Section B.
Both sections are worth equal marks.

Section A

A1 a) The threshold of sensitivity of the human eye is approximately 100 photons per second at a wavelength of 500 nm. Determine this threshold as an incident power.

b) What is the energy in eV of photons at each end of the visible spectrum, from, say 380 nm to 770 nm?

c) What is the minimum vertical size that a mirror has to be, and where should it be placed, for a standing person to be able to see their feet and the top of their head in the mirror? Justify your conclusion.

A2. A light wave has an electric field given by (all values are given in S.I. units):

$$\vec{\mathbf{E}}(z,t) = 870\,\hat{\mathbf{x}}\cos\left(9.52\times10^{6}\,z - 2.86\times10^{15}\,t\right) + 870\,\hat{\mathbf{y}}\sin\left(9.52\times10^{6}\,z - 2.86\times10^{15}\,t\right).$$

Determine the wavelength, the wave vector, the period and velocity (in appropriate units). With the aid of a diagram, carefully describe the polarization of this wave.

A3. Two monochromatic linearly-polarized light beams traveling in opposite directions along the z-axis, and with the same amplitude, speed, frequency and polarization, overlap in some region of space. By representing the two electric fields of these waves in terms of *complex* fields $\tilde{\mathbf{E}}_1(z,t)$ and $\tilde{\mathbf{E}}_2(z,t)$, find the temporal and spatial dependence of the resulting (*real*) electric field $\vec{\mathbf{E}}_{\mathrm{tot}}(z,t)$. Describe the behaviour of $\vec{\mathbf{E}}_{\mathrm{tot}}(z,t)$ at various times and locations.

A4. A pulsed laser operating at a central wavelength of 800 nm with pulse duration of 10 fs and pulse energy of 1 mJ emits a beam 5 mm in diameter which passes through a converging lens to form a spot with diameter 100 μm. Estimate the peak amplitude of the electric field a) directly after exiting the laser, and b) at the focal point of the lens.

A5. a) Consider light with frequency ν_0 in the laboratory frame traveling along the x-axis. Write down the relationship between the observed frequency of the light in a molecule’s frame of reference when the molecule is traveling with a speed v_x along the x-axis.

b) The probability distribution function of molecular speeds along the x-axis, $P(v_x)$, in a gas at temperature T is:

$$P(v_x) \propto e^{\frac{-\,m v_x^{2}}{2kT}}$$

Calculate the linewidth, Δv in MHz, of the absorption profile for the $^{12}C^{16}O_2$ isotope on the 10.6 μm transition, assuming the gas temperature is 373 K.

A6. Calculate the ratio of spontaneous to stimulated emission for light of wavelength 0.5 μm emitted from a blackbody at 3000 K.

Section B

B1. When a light wave passes through a non-magnetic, dielectric material the wave equation satisfied by the complex electric field is given by:

$$\nabla^2 \tilde{\mathbf{E}} = \left(\frac{1+\tilde{\chi}}{c^2} \right) \frac{\partial^2 \tilde{\mathbf{E}}}{\partial t^2},$$

where c is the speed of light in vacuum.

a) What is meant by the *complex susceptibility*, $\tilde{\chi}$, and how does this relate to the *complex refractive index*, $\tilde{n}$? By assuming a wave of the form:

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(\tilde{k}z - \omega t)},$$

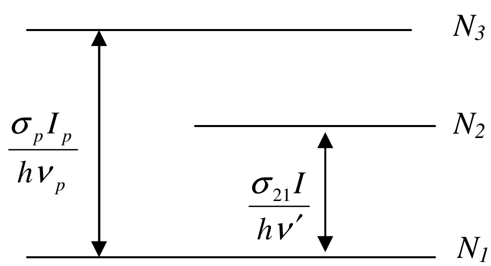
find a relationship between $\tilde{k}$ and $\tilde{n}$.

b) Suppose that a wave packet is composed of light with a narrow range of frequencies for which the absorption by the medium is negligible, but the real part of the refractive index has an observable dependence on frequency: $n_R(\omega)$. Briefly describe what is meant by the *group velocity* of this wave packet, and state why it is a useful property in optics. Show that the group velocity is given by:

$$v_g = \frac{v_p}{1 + \frac{\omega_p v_p}{c} \frac{dn_R(\omega)}{d\omega}},$$

where v_p is the phase velocity of the central harmonic wave, with angular frequency ω_p.

B2. The first laser to be built was the ruby laser, by Theodore Maiman in 1960, which is a three-level laser system. A partial energy level diagram of this, depicting **only** the stimulated emission and stimulated absorption processes, is shown below:



a) Complete this diagram, including **all** relevant processes.

b) Write down rate equations for each of N_1, N_2 and N_3; the numbers of atoms per unit volume in each level.

c) Assume the system to be closed: i.e. $N_T = N_1 + N_2 + N_3 =$ constant. Find the **small-signal** ($I \approx 0$), **steady-state** values of N_1 and N_2 in terms of N_T, assuming that the decay from $3 \rightarrow 1$ is negligible.

d) Find the conditions under which there will be a small-signal steady-state population inversion.

B3. a) A parallel-mirror He-Ne laser which has mirrors with reflectances of 1 and 0.97, and are separated by 0.22 m, is pumped such that its peak small-signal gain coefficient, γ_0, is 2×10^{-3} cm^{-1}. Given that a He-Ne laser has an inhomogeneously broadened linewidth of 1.5 GHz, and assuming that $n \approx 1$, find a) the maximum, and b) the minimum number of modes that can be emitted from this laser at one time.

b) Give a brief description (a few sentences each and/or a diagram) of four different types of lasers.

Potentially useful equations:

$$\rho(\nu) = \frac{8\pi\upsilon^2}{c^3}\frac{h\upsilon}{e^{\frac{h\nu}{kT}} - 1}$$

$$A_{21} = \frac{8\pi h\nu_0^{\;3}}{c^3}B_{21}$$

$$g(\nu) = \frac{\alpha}{4\pi^2(\nu-\nu_0)^2 + \frac{\alpha^2}{4}} \quad ; \quad \Delta\nu = \frac{\alpha}{2\pi}$$

$$g(\nu) = \sqrt{\frac{\alpha}{\pi}}\,e^{-\alpha(\nu-\nu_0)^2} \quad ; \quad \Delta\nu = 2\sqrt{\frac{\ln(2)}{\alpha}}$$

$$\gamma_{th} = \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

$$\gamma_{th} = \frac{1}{L}\ln\left(\frac{1}{R_1R_2R_3}\right)$$