Key Formulas from *Pedrottis' Introduction to Optics*, 4th ed.

Ch. 4 WAVES

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{\upsilon^2}\frac{\partial^2 y}{\partial t^2} \qquad (4.3)$$

$$\nabla^2\tilde{\psi} = \frac{1}{\upsilon^2}\frac{\partial^2\tilde{\psi}}{\partial t^2} \qquad (4.25)$$

$$\tilde{\psi}\left(\vec{\mathbf{r}},t\right) = \left(\frac{A}{r}\right)e^{i(kr-\omega t)} \qquad (4.26)$$

$$\tilde{\psi}\left(\rho,z,t\right) = \frac{A}{\sqrt{\rho}}e^{i(k\rho-\omega t)} \qquad (4.27)$$

$$u\left(\vec{\mathbf{r}},t\right) = \varepsilon_0 E\left(\vec{\mathbf{r}},t\right)^2 \quad \text{in free space} \qquad (4.39)$$

$$\vec{\mathbf{S}}\left(\vec{\mathbf{r}},t\right)\frac{1}{\mu_0}\left(\vec{\mathbf{E}}\left(\vec{\mathbf{r}},t\right)\times\vec{\mathbf{B}}\left(\vec{\mathbf{r}},t\right)\right) \quad \text{in free space} \qquad (4.44)$$

$$I\left(\vec{r}\right) = <\left|\vec{\mathbf{S}}\left(\vec{\mathbf{r}},t\right)\right|> = \varepsilon_0 c < E\left(\vec{\mathbf{r}},t\right)^2 > \quad \text{in free space} \qquad (4.45)$$

$$I = \frac{1}{2}\varepsilon_0 c^2 E_0 B_0 = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\left(\frac{c}{\mu_0}\right)B_0^2 \qquad (4.46)$$

for a single harmonic wave in free space
(for a linear material $\varepsilon_0 \rightarrow n^2\varepsilon_0$ and $c \rightarrow c/n$)

$$I = \frac{1}{2}\varepsilon_0 c\left\langle\tilde{\mathbf{E}}\left(\vec{\mathbf{r}},t\right)\tilde{\mathbf{E}}^*\left(\vec{\mathbf{r}},t\right)\right\rangle \qquad (4.49)$$

$$\frac{\lambda'}{\lambda} \approx 1-\frac{\upsilon}{c} \qquad (4.53)$$

Ch. 6 SUPERPOSITION OF WAVES

$$E_R \equiv E_R\left(P,t\right) = E_0\cos\left(\alpha-\omega t\right)$$

where $E_0^2 = \left(\sum_{i=1}^{N}E_{0i}\sin\alpha_i\right)^2 + \left(\sum_{i=1}^{N}E_{0i}\cos\alpha_i\right)^2$

or, equivalently $E_0^2 = \sum_{i=1}^{N}E_{0i}^2 + 2\sum_{j>i}^{N}\sum_{i=1}^{N}E_{0i}E_{0j}\cos\left(\alpha_j-\alpha_i\right)$ (6.14-6.16)

and $\tan\alpha = \dfrac{\sum_{i=1}^{N}E_{0i}\sin\alpha_i}{\sum_{i=1}^{N}E_{0i}\cos\alpha_i}$

$$\upsilon_g = \left.\frac{c}{n+\omega\dfrac{dn(\omega)}{d\omega}}\right|_{\omega_{\text{pk}}} = \left.\frac{\upsilon_p}{1+\dfrac{\omega}{n}\dfrac{dn(\omega)}{d\omega}}\right|_{\omega_{\text{pk}}} \quad (6.48)$$

$$= \left.\frac{c}{n-\lambda_0\dfrac{dn(\lambda_0)}{d\lambda_0}}\right|_{\lambda_{0,\text{pk}}} = \left.\frac{\upsilon_p}{1-\dfrac{\lambda_0}{n}\dfrac{dn(\lambda_0)}{d\lambda_0}}\right|_{\lambda_{0,\text{pk}}} \quad (6.49)$$

Ch. 13 OPTICAL FIBERS AND COMMUNICATIONS TECHNOLOGY

$$\mathcal{R}\ (\text{in lines/mm}) \approx \frac{500}{d\left(\text{in } \mu\text{m}\right)} \quad (13.1)$$

$$\text{NA} \equiv n_0 \sin\theta_{\max} = n_1 \sin\theta'_{\max} \quad (13.5)$$

$$L_{\text{s}} = d\sqrt{\left(\frac{n_1}{n_0 \sin\theta}\right)^2 - 1} \quad (13.8)$$

$$m_x \cong \frac{2d\, n_1 \cos\phi_{m_x}}{\lambda} \qquad \text{with } m_x \text{ an integer} \quad (13.11)$$

$$m_{x,\max} \cong \frac{2d\, n_1 \cos\phi_{\text{c}}}{\lambda} = \frac{2d}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2d}{\lambda}\,\text{NA} \quad (13.12)$$

$$\text{total \# of modes}_{\text{sq}} \cong 8\left(\frac{d}{\lambda}\,\text{NA}\right)^2 \quad (13.13)$$

$$\text{total \# of modes}_{\text{cyl}} \approx 4\left(\frac{d}{\lambda}\,\text{NA}\right)^2 \quad (13.14)$$

$$d < \frac{2.4\lambda}{\pi(\text{NA})} \quad (13.15)$$

$$I_L = I_0 \exp\left(-\alpha L_{\text{km}}\right) \quad \Rightarrow \quad \alpha = \frac{1}{L_{\text{km}}}\ln\left(\frac{I_0}{I_L}\right) \quad (13.16)$$

$$I_L = I_0 \cdot 10^{-\left(\alpha_{dB} L_{\text{km}}/10\right)} \quad \Rightarrow \quad \alpha_{\text{dB}} = \frac{10}{L_{\text{km}}}\log\left(\frac{I_0}{I_L}\right) \quad (13.18)$$

$$\left(\frac{\delta\tau}{L}\right)_{\text{step-index}} = \frac{n_1}{c}\left(\frac{n_1 - n_2}{n_2}\right) \quad (13.19)$$

$$\left(\frac{\delta\tau}{L}\right)_{\text{GRIN}} \approx \frac{n_1}{2c}\Delta_n^{\,2} \quad ; \quad \Delta_n = \left(n_1^2 - n_2^2\right)/\,2n_1^2 \quad (13.21)$$

$$\left(\frac{\delta\tau}{L}\right) = -\frac{\lambda_{\text{pk}}}{c}\frac{d^2 n(\lambda)}{d\lambda^2}\bigg|_{\lambda=\lambda_{\text{pk}}} \times \Delta\lambda \equiv -M\,\Delta\lambda \qquad (13.25)$$

$$\left(\frac{\delta\tau}{L}\right) = -\frac{\lambda_{\text{pk}}}{c}\frac{d^2 n_{\text{eff}}\left(\lambda_{\text{pk}}\right)}{d{\lambda_{\text{pk}}}^2} \times \Delta\lambda \equiv -M'\,\Delta\lambda \qquad (13.27)$$

$$\left|\frac{\delta\tau}{L}\right| \approx \frac{1}{c}\left|n_{\text{eff},x} - n_{\text{eff},y}\right| \qquad (13.28)$$

Ch. 14 MATHEMATICAL TREATMENT OF POLARIZATION

$$\mathbf{R}(\theta) \equiv \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad (14.5)$$

$$\mathbf{V}_{\text{norm}} = \frac{1}{\sqrt{A^2+B^2+C^2}}\begin{bmatrix} A \\ B+iC \end{bmatrix}$$ elliptical, counter – clockwise if $A, C > 0$

and then $\tan 2\psi = \dfrac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2}$

where $E_{0x} = A,\ E_{0y} = \sqrt{B^2+C^2}$, and $\varepsilon = \Delta\varphi = \tan^{-1}\left(\dfrac{C}{B}\right)$ (14.11) – (14.13); (p. 523)

and $\left(\dfrac{E_x}{E_{0x}}\right)^2 + \left(\dfrac{E_y}{E_{0y}}\right)^2 - 2\left(\dfrac{E_x}{E_{0x}}\right)\left(\dfrac{E_y}{E_{0y}}\right)\cos\ \varepsilon = \sin^2\varepsilon$

if optical cmpt is rotated *clockwise* $\quad \mathbf{M}' = \mathbf{R}(\theta)\mathbf{M}\,\mathbf{R}^{\mathrm{T}}(\theta)$ (p. 530)

$$\begin{aligned} S_0 &= I_{\text{tot}} \\ S_1 &= I_{\leftrightarrow} - I_{\updownarrow} \\ S_2 &= I_{\nearrow\!\!\!\swarrow} - I_{\nwarrow\!\!\!\searrow} \\ S_3 &= I_{\mathfrak{R}} - I_{\mathfrak{L}} \end{aligned} \qquad (14.27)$$

$$\begin{aligned} S_0 &= E_{0x}^2 + E_{0y}^2 = E_0^2 \\ S_1 &= E_{0x}^2 - E_{0y}^2 \\ S_2 &= 2E_{0x}E_{0y}\cos\left(\varphi_y - \varphi_x\right) \\ S_3 &= -2E_{0x}E_{0y}\sin\left(\varphi_y - \varphi_x\right) \end{aligned} \quad \text{or} \quad \begin{aligned} S_0 &= \left|\tilde{E}_{0x}\right|^2 + \left|\tilde{E}_{0y}\right|^2 = \left|\tilde{\mathbf{E}}_0\right|^2 \\ S_1 &= \left|\tilde{E}_{0x}\right|^2 - \left|\tilde{E}_{0y}\right|^2 \\ S_2 &= 2\,\mathrm{Re}\left(\tilde{E}_{0x}\tilde{E}_{0y}^*\right) \\ S_3 &= 2\,\mathrm{Im}\left(\tilde{E}_{0x}\tilde{E}_{0y}^*\right) \end{aligned} \qquad (14.28)$$

$$p \equiv \left({S_1}^2 + {S_2}^2 + {S_3}^2\right)^{1/2} / S_0 \qquad (14.34)$$

Ch. 15 POLARIZATION IN PRACTICE

$$p(\theta) = \frac{\sin^2\theta}{1+\cos^2\theta} \qquad (15.3)$$

$$\Delta\varphi = 2\pi\left(\frac{\Delta}{\lambda_0}\right) = \left(\frac{2\pi}{\lambda_0}\right)\left|n_{\perp} - n_{\parallel}\right|\,d \qquad (15.4)$$

$\beta = \alpha_\lambda L$ (if sample is solid; L usually in mm)
$\beta = \alpha_\lambda L\rho$ (if sample is liquid; L usually in dm) (15.5)
$\beta = \alpha_\lambda L c_m$ (if sample is solution; L usually in dm)

$$\beta = \frac{\pi L}{\lambda_0}\left(n_{\mathfrak{L}} - n_{\mathfrak{R}}\right) \quad (15.10)$$

Ch. 16 LIGHT-MATTER INTERACTIONS

$$\Gamma_{\text{abs}} \equiv \sigma_{\text{abs}}(\nu)F \quad (16.1)$$

$$I_L = I_0 e^{-N\sigma_{\text{abs}} L} \quad (16.4)$$

$$\sigma_{12}(\nu') = \frac{B_{12} g(\nu') h\nu'}{c} \quad ; \quad \sigma_{21}(\nu') = \frac{B_{21} g(\nu') h\nu'}{c} \quad (16.9); (16.10)$$

$$A_{21} = \frac{8\pi h\nu_{12}^3}{c^3} B_{21} \quad ; \quad g_1 B_{12} = g_2 B_{21} \quad (16.18); (16.19)$$

$$N_{\text{inv}} \equiv N_2 - \frac{g_2}{g_1} N_1 \quad (16.23)$$

$$\rho(\nu_{12}) = \frac{8\pi h\nu_{12}^3}{c^3} \frac{1}{\exp\left(h\nu_{12}/k_B T\right) - 1} \quad (16.25)$$

$$\frac{dI}{dz} = \sigma_{21}(\nu')\left(N_2 - \left(\frac{g_2}{g_1}\right)N_1\right) I \equiv \gamma(I,\nu')I \quad (16.35)$$

$$g_L(\nu) = \frac{\Delta\nu_L}{2\pi\left(\left(\nu - \nu_{12}\right)^2 + \Delta\nu_L{}^2/4\right)} \quad (16.37)$$

$$g_G(\nu) = \sqrt{\frac{4\ln 2}{\pi\,\Delta\nu_G{}^2}} \exp\left(-\frac{4\ln 2\left(\nu - \nu_{12}\right)^2}{\Delta\nu_G{}^2}\right) \quad (16.38)$$

$$\Delta\nu_{\tau_2} = \frac{1}{2\pi}\left(\frac{1}{\tau_2}\right) \quad ; \quad \Delta\nu_p = \frac{1}{2\pi}\left(\frac{2}{\overline{\tau}_{\text{coll}}}\right) \quad (16.43); (16.44)$$

$$\Delta\nu_{\text{D}} = \frac{2\nu_{12}}{c}\sqrt{\frac{2k_B T\ln(2)}{M}} \approx 7.16\times 10^{-7}\,\nu_{12}\sqrt{\frac{T}{M_{\text{amu}}}} \quad (16.46)$$

Ch. 17 LASERS AND LASER OPERATION

$$\ell_t = c\tau_c = \frac{c}{\Delta\nu} \qquad (17.2)$$

$$\theta_{\text{FF}} = \frac{\lambda}{\pi w_0} \approx 0.318\frac{\lambda}{w_0} \qquad (17.3)$$

$$R_{p1} \equiv \frac{\kappa_{31}}{(\kappa_{30}+\kappa_{31}+\kappa_{32})}\left(\frac{\sigma_p I_p}{h\nu_p}N_T\right) \qquad (\text{p.623})$$

$$R_{p2} \equiv \frac{\kappa_{32}}{(\kappa_{30}+\kappa_{31}+\kappa_{32})}\left(\frac{\sigma_p I_p}{h\nu_p}N_T\right) \qquad (\text{p.623})$$

$$\gamma_{ss,\text{homogeneous}}(I,\nu') = \underbrace{\frac{\sigma R_{p2}/\kappa_2}{1+(\sigma I/h\nu')/\kappa_2}}_{\text{ideal 4-level gain medium only}} \equiv \underbrace{\frac{\gamma_0}{1+I/I_S}}_{\text{generally true}} \qquad (17.14)$$

$$\gamma_0(\nu',I_p) \equiv \frac{\sigma(\nu')R_{p2}(I_p)}{\kappa_2} = \sigma(\nu')R_{p2}(I_p)\tau_2 \qquad (17.15)$$

$$I_{\text{sat}}(\nu') \equiv \frac{h\nu'\kappa_2}{\sigma(\nu')} = \frac{h\nu'}{\sigma(\nu')\tau_2} \qquad (17.16)$$

$$I_{\text{out,ss}} = T_3 I_{\text{sat}}\left(\frac{\gamma_0 L - \ln\frac{1}{S}}{1-S}\right) \qquad (17.20)$$

$$I_{\text{out,ss}} = \frac{T_2 I_{\text{sat}}}{2}\frac{\gamma_0(2L) - \ln(1/R_1R_2)}{\left(1-\sqrt{R_1R_2}\right)\left(1+\sqrt{R_2/R_1}\right)} \qquad (17.24)$$

$$\gamma_{\text{ss,inhomogeneous}} = \frac{\gamma_0}{\sqrt{1+I/I_{\text{sat}}}} \qquad (17.26)$$

$$Q \equiv 2\pi\left(\frac{\text{energy } \textit{stored} \text{ in a cavity}}{\text{energy } \textit{loss} \text{ per cycle}}\right) \qquad (17.29)$$

APPENDICES

A. PHYSICAL CONSTANTS (generally given here to 3 or 4 sig. figs)

$\Delta\nu_{\text{Cs}} \equiv 9\ 192\ 631\ 770$ Hz

$c \equiv 299\,792\,458$ m/s

$h = 6.63\times10^{-34}$ Js ; $\hbar = 1.055\times10^{-34}$ Js

$e = 1.602 \times 10^{-19}\ \text{C}$

$N_{\text{A}} = 6.022 \times 10^{23}\ \text{mol}^{-1}$

$k_B = 1.381 \times 10^{-23}\ \text{JK}^{-1}$

$K_{cd} \equiv 683\ \text{lmW}^{-1}$

$eV \equiv 1.602 \times 10^{-19}\ \text{J}$

$m_e = 9.109 \times 10^{-31}\ \text{kg}$

$R = 8.314\ \text{J mol}^{-1}\text{K}$

$\sigma = 5.670 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$

$\text{u} = 1.660 \times 10^{-27}\ \text{kg}$

$R_\infty = 1.097 \times 10^{7}\ \text{m}^{-1}$

$\mu_0 = 1.257 \times 10^{-6}\ \text{NA}^{-2} \approx 4\pi \times 10^{-7}\ \text{NA}^{-2}$

$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{Fm}^{-1}$

$m_{\text{e}} = 9.11 \times 10^{-31}\ \text{kg}$

$m_{\text{p}} = 1.67 \times 10^{-27}\ \text{kg}$

$a_0 = 5.29 \times 10^{-11}\ \text{m}$

$\mu_{\text{B}} = 9.27 \times 10^{-24}\ \text{J T}^{-1}$

B. MATHEMATICAL FORMULAS

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\sin A \pm \sin B = 2\sin\left[\tfrac{1}{2}(A \pm B)\right]\cos\left[\tfrac{1}{2}(A \mp B)\right]$$

$$\cos A + \cos B = 2\cos\left[\tfrac{1}{2}(A + B)\right]\cos\left[\tfrac{1}{2}(A - B)\right]$$

$$\cos A - \cos B = -2\sin\left[\tfrac{1}{2}(A + B)\right]\sin\left[\tfrac{1}{2}(A - B)\right]$$