Formula Sheet

(When quoting formulae from this sheet, write down **the** formula **and** its number)

1. LIGHT

$$\left.\begin{aligned} &\vec{\nabla}\cdot\vec{E}(\vec{r},t)=\frac{\rho(\vec{r},t)}{\varepsilon_0} \qquad \vec{\nabla}\times\vec{E}(\vec{r},t)=-\frac{\partial\vec{B}(\vec{r},t)}{\partial t} \\ &\vec{\nabla}\cdot\vec{B}(\vec{r},t)=0 \qquad \vec{\nabla}\times\vec{B}(\vec{r},t)=\mu_0\vec{J}(\vec{r},t)+\mu_0\varepsilon_0\frac{\partial\vec{E}(\vec{r},t)}{\partial t} \end{aligned}\right\} \quad (1.1)$$

$$\vec{D}(\vec{r},t)=\varepsilon_0\vec{E}(\vec{r},t)+\vec{P}(\vec{r},t) \quad (1.2)$$

$$\vec{H}(\vec{r},t)=\frac{\vec{B}(\vec{r},t)}{\mu_0}-\vec{M}(\vec{r},t) \quad (1.3)$$

$$\vec{B}(\vec{r},t)=\frac{\vec{k}\times\vec{E}(\vec{r},t)}{\omega} \quad (1.4)$$

$$u_{em}(\vec{r},t)=\frac{1}{2}\left(\varepsilon_0 E(\vec{r},t)^2+\frac{1}{\mu_0}B(\vec{r},t)^2\right) \quad (1.5)$$

$$\vec{S}(\vec{r},t)=\frac{1}{\mu_0}\left(\vec{E}(\vec{r},t)\times\vec{B}(\vec{r},t)\right) \quad (1.6)$$

$$I(\vec{r})=<\left|\vec{S}(\vec{r},t)\right|> \quad (1.7)$$

$$f(z)=\int_{-\infty}^{\infty}\tilde{g}(k)e^{ikz}dk\,; \quad \tilde{g}(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(z)e^{-ikz}dz \quad (1.8)$$

$$\text{transition strength}\propto\left|<\gamma'\ j'm'|\,\vec{d}\cdot\vec{E}(\vec{r},t)|\gamma\ j\ m>\right|^2 \quad (1.9)$$

$$v_g=v_p\left(1+\frac{\omega_p}{n(\omega_p)}\frac{dn(\omega)}{d\omega}\right)_{\omega_p}^{-1} \quad (1.10)$$

$$v_g=v_p\left(1-\frac{k_p}{n(k_p)}\frac{dn(k)}{dk}\right)_{k_p} \quad (1.11)$$

$$v_g=v_p\left(1+\frac{\lambda_p}{n(\lambda_p)}\frac{dn(\lambda)}{d\lambda}\right)_{\lambda_p} \quad (1.12)$$

$$I_P=I_1+I_2+2\sqrt{I_1I_2}\cos\delta \quad (1.13)$$

$$\tilde{\Gamma}_{12}(\tau)=<\tilde{E}_1(t+\tau)\tilde{E}_2^{\,*}(t)> \quad (1.14)$$

$$\tilde{\gamma}_{12}(\tau)=\frac{<\tilde{E}_1(t+\tau)\tilde{E}_2^{\,*}(t)>}{\sqrt{<\left|\tilde{E}_1\right|^2><\left|\tilde{E}_2\right|^2>}} \quad (1.15)$$

$$I_P=I_1+I_2+2\sqrt{I_1I_2}\,\mathrm{Re}\left[\tilde{\gamma}_{12}(\tau)\right] \quad (1.16)$$

$$V=\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}} \quad (1.17)$$

2. INTERACTION OF LIGHT AND MATTER

$$\vec{p}=\alpha\,\vec{E}_{local} \quad (2.1)$$

$$\vec{P}(\vec{r})=\varepsilon_0\chi_e\vec{E}(\vec{r}) \quad (2.2)$$

$$\chi_e=\frac{N\alpha/\varepsilon_0}{1-N\alpha/3\varepsilon_0} \quad (2.3)$$

$$K=1+\chi_e \quad (2.4)$$

$$\tilde{p}(\omega)=\tilde{\alpha}\,\tilde{E}_{local}(\omega) \quad (2.5)$$

$$\tilde{\alpha}(\omega)=\frac{q^2}{m}\left(\frac{1}{\left(\omega_0'^2-\omega^2\right)-i\gamma\omega}\right) \qquad \omega_0'=\sqrt{\frac{k}{m}} \quad (2.6)$$

$$\tilde{P}(\omega)=\varepsilon_0\tilde{\chi}_e(\omega)\tilde{E}(\omega) \quad (2.7)$$

$$\tilde{\chi}_e(\omega)=\frac{q^2N}{m\varepsilon_0}\left(\frac{1}{\left(\omega_0^{\,2}-\omega^2\right)-i\gamma\omega}\right) \qquad \omega_0^{\,2}=\omega_0'^2-\frac{Nq^2}{3m\varepsilon_0} \quad (2.8)$$

$$\tilde{K}=1+\tilde{\chi}_e \quad (2.9)$$

$$\tilde{n}(\omega)=\frac{c\tilde{k}}{\omega} \quad (2.10)$$

$$\vec{E}^{\parallel}_{medium\ 1}=\vec{E}^{\parallel}_{medium\ 2} \quad (2.11)$$

$$B^{\perp}_{away,medium\,1}=B^{\perp}_{towards,medium\,2} \quad (2.12)$$

$$E^{\perp}_{away,medium\,1}-E^{\perp}_{towards,medium\,2}=\frac{\sigma}{\varepsilon_0} \quad (2.13)$$

$$\vec{H}^{\parallel}_{medium\ 1}-\vec{H}^{\parallel}_{medium\ 2}=\vec{K}_{free}\times\hat{n}_{2\to1} \quad (2.14)$$

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$$I_t = \frac{I_i}{1+\frac{4r^2}{\left(1-r^2\right)^2}\sin^2\frac{\delta}{2}} \qquad \delta = \frac{2\pi}{\lambda_0}2n_2 d\cos\theta_t \tag{2.15}$$

$$\Im = \frac{\text{fringe separation}}{\text{FWHM}} = \frac{\pi r}{1-r^2} \tag{2.16}$$

$$\Delta\nu_{fsr} = \frac{c}{2n_2 d} \qquad \Delta\lambda_{fsr} \approx \frac{\lambda^2}{2n_2 d} \tag{2.17}$$

$$\underline{\underline{M}} = \begin{bmatrix} \cos\frac{\delta}{2} & \frac{i\sin\frac{\delta}{2}}{n_1} \\ in_1\sin\frac{\delta}{2} & \cos\frac{\delta}{2} \end{bmatrix} \tag{2.18}$$

$$\tilde{r} \equiv \frac{\tilde{E}_{r1}}{\tilde{E}_i} = \frac{n_0 M_{11} + n_0 n_s M_{12} - M_{21} - n_s M_{22}}{n_0 M_{11} + n_0 n_s M_{12} + M_{21} + n_s M_{22}} \tag{2.19}$$

$$\tilde{k} \approx \sqrt{\frac{\mu_0\sigma\omega}{2}}(1+\mathrm{i}) \qquad \sigma >> \omega\varepsilon_0 \tag{2.20}$$

$$R \approx 1 - 2\sqrt{\frac{2\omega\varepsilon_0}{\sigma}} \qquad \sigma >> \omega\varepsilon_0 \tag{2.21}$$

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1} \qquad \rho(\lambda) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \tag{2.22}$$

$$g(\nu) = \frac{\alpha}{4\pi^2(\nu-\nu_0)^2+\frac{\alpha^2}{4}} \qquad \Delta\nu = \frac{\alpha}{2\pi} \tag{2.23}$$

$$g(\nu) = \sqrt{\frac{\alpha}{\pi}}e^{-\alpha(\nu-\nu_0)^2} \qquad \Delta\nu = 2\sqrt{\frac{\ln 2}{\alpha}} \tag{2.24}$$

$$\Delta\nu_D = 7.16\times10^{-7}\,\nu_0\sqrt{\frac{T}{M_{amu}}} \tag{2.25}$$

3. LASER OPERATION AND APPLICATIONS

$$g_1 B_{12} = g_2 B_{21} \qquad A_{21} = \frac{8\pi h\nu_0^3}{c^3}B_{21} \tag{3.1}$$

$$\sigma_{21}(\nu') \equiv \frac{B_{21}g(\nu')h\nu'}{c} \tag{3.2}$$

$$\frac{dI}{dz} = \sigma(N_2 - N_1)I \equiv \gamma(I,\nu')I \tag{3.3}$$

$$\gamma(I,\nu') = \frac{\gamma_0(\nu')}{1+\frac{\sigma(\nu')I}{h\nu' K_2}} \equiv \frac{\gamma_0(\nu')}{1+\frac{I}{I_s(\nu')}} \tag{3.4}$$

$$I_{out,ss} = T_3 I_s\left(\frac{\gamma_0 L - \ln\frac{1}{S}}{1-S}\right) \tag{3.5}$$

$$I_{out,ss} = \frac{T_2 I_s}{2}\frac{2\gamma_0 L - \ln\frac{1}{R_1 R_2}}{\left(1-\sqrt{R_1 R_2}\right)\left(1+\sqrt{R_2/R_1}\right)} \tag{3.6}$$

$$W_{q_1,c} = \frac{{q_1}^2\pi^2\hbar^2}{2m_c{d_1}^2} + \frac{\hbar^2 k^2}{2m_c} \tag{3.7}$$

4. MATHEMATICAL FORMULAE

$$\sin(2\theta) = 2\sin\theta\cos\theta \tag{4.1}$$

$$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B \tag{4.2}$$

$$\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B \tag{4.3}$$

$$\sin A \pm \sin B = 2\sin\left[\tfrac{1}{2}(A\pm B)\right]\cos\left[\tfrac{1}{2}(A\mp B)\right] \tag{4.4}$$

$$\cos A + \cos B = 2\cos\left[\tfrac{1}{2}(A+B)\right]\cos\left[\tfrac{1}{2}(A-B)\right] \tag{4.5}$$

$$\cos A - \cos B = -\sin\left[\tfrac{1}{2}(A+B)\right]\sin\left[\tfrac{1}{2}(A-B)\right] \tag{4.6}$$

5. FUNDAMENTAL CONSTANTS

$c \equiv 299\,792\,458$ m/s

$e = 1.602\times10^{-19}$ C

$m_e = 9.109\times10^{-31}$ kg

$1\,\text{amu} = 1.661\times10^{-27}$ kg

$\varepsilon_0 = 8.85\times10^{-12}$ Fm^{-1}

$\mu_0 \equiv 4\pi\times10^{-7}$ NA^{-2}

$h = 6.626\times10^{-34}$ Js ; $\qquad \hbar = 1.055\times10^{-34}$ Js

$k_B = 1.381\times10^{-23}$ JK^{-1}

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