Midterm: Tuesday 2 March 2010
Time allowed: 1 hour 50 mins
Formula sheets provided

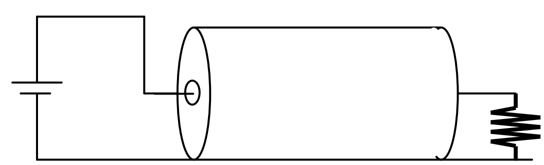
1. Describe what is meant by the term *local conservation of charge*. Derive an expression for this from Maxwell's equations.

2. The flux of $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ out of an arbitrary closed surface in free space can be shown to satisfy:

$$\Phi_{\vec{S}} = -\frac{d}{dt}\int \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)d\tau$$

a) With the help of a diagram and a mathematical expression, explain what is meant by the *flux of a vector field out of a closed surface*? Explain what this equation means in words.
b) Rewrite the equation above in differential form and explain in words what *this* means.

3. The conductors of a coaxial cable, with an inner conductor of outer radius s_1, and an outer conductor of inner radius s_2, are connected to a battery and resistor of resistance R, as shown on the right.



The electric and magnetic fields between $s = s_1$ and $s = s_2$ are given by:

$$\vec{E}(\vec{r}) = \frac{V}{r\ln\frac{s_2}{s_1}}\hat{s} \qquad \vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi r}\hat{\phi},$$

Find the Poynting vector in the gap, and show that the power flowing to the resistor is V^2/R.

4. Consider a complex, plane, electromagnetic wave in free space of the form:

$$\tilde{\boldsymbol{E}}(z,t) = \tilde{\boldsymbol{E}}_0 e^{i(kz-\omega t)} \quad \tilde{\boldsymbol{B}}(z,t) = \tilde{\boldsymbol{B}}_0 e^{i(kz-\omega t)}$$

Derive the vector relation between $\tilde{\boldsymbol{E}}_0$ and $\tilde{\boldsymbol{B}}_0$.

5. Given a hollow copper rectangular cuboid of dimensions $1\times2\times3\ \text{cm}^3$,
a) What are the general boundary conditions at each surface?
b) Guess a reasonable, trial, solution for $\vec{E}(\vec{r},t)$ inside the box composed of sines and cosines in both space and time. It should satisfy these boundary conditions on one particular surface, and show this to be the case.
c) How many electromagnetic modes of wavelength λ are there with $\lambda \geq (4/\sqrt{5})$ cm ?