Physics 421H – Electromagnetic Waves. Midterm: Monday 7th November 2005

Answer all three questions. Show your working.

Allowed: 1 hour. Calculator, Formula Sheet (given), and up to 1 side of 8 ½×11 paper (written by you).

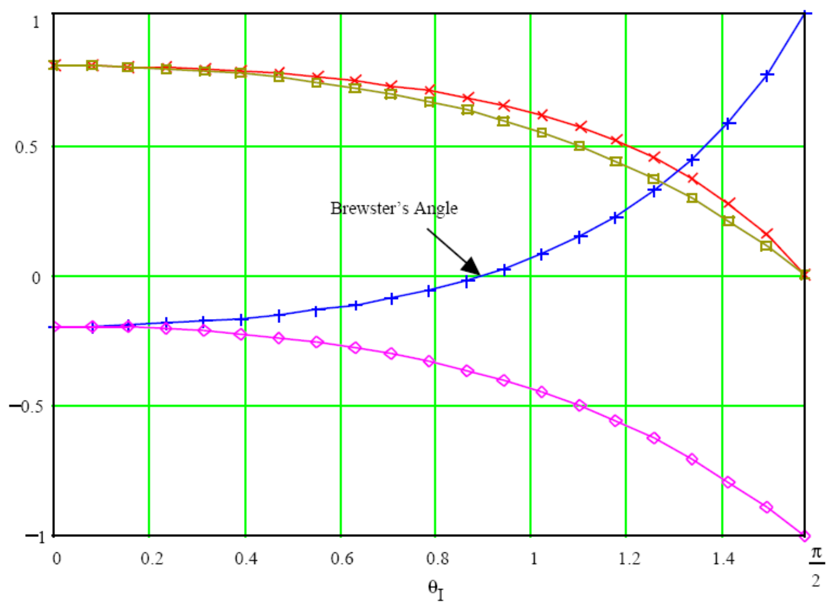
1. A long coaxial cable with one end at $z = 0$ carries current I down the inner cylinder held at potential V (radius a), and back down the outer cylinder held at zero potential (with inner radius b). A resistor, R, is placed across the cylinders at the end of the cable at $z = z_0$.

(i) Find the electric and magnetic fields in the region between the cylinders.

(ii) Find the total energy per unit time transported by the fields. In which direction does it point?

2. (i) Write down (or derive if necessary) the general boundary conditions (in terms of ***E***, ***B***, ***D***, ***H***) at an interface between any two media. To what do these simplify at a boundary between non-conducting, non-magnetic, uncharged linear isotropic materials with $\varepsilon_r \neq 1$?

(ii) A monochromatic plane wave exhibits external reflection at angle θ_I on an air-plastic boundary. The magnitude of its transmitted and reflected electric fields as a function of θ_I is shown below:



(From http://www.physics.ubc.ca/~mattison/Courses/Phys454/lecture20.pdf)

(iii) Describe what the four traces (i.e. the vertical and diagonal crosses and squares) correspond to. Determine the refractive index of the plastic.

3. (i) Starting with Maxwell's equations, show that the electric field in vacuum satisfies the 3-D wave equation.

(ii) The electric field of a wave in vacuum is given by $\mathbf{E}(\rho, t) = \mathbf{E_0} f(\rho\varpi + v^2 t)$, where $\mathbf{E_0}$ is independent of x, y, z and t and f is some known function. Find the corresponding magnetic field $\mathbf{B}(\rho, t)$ and the electromagnetic energy density as a function of space and time.

Formula Sheet

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$
$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$
$$\sin(A) \pm \sin(B) = 2\sin\left[\tfrac{1}{2}(A \pm B)\right]\cos\left[\tfrac{1}{2}(A \mp B)\right]$$
$$\cos(A) + \cos(B) = 2\cos\left[\tfrac{1}{2}(A + B)\right]\cos\left[\tfrac{1}{2}(A - B)\right]$$
$$\cos(A) - \cos(B) = -2\sin\left[\tfrac{1}{2}(A + B)\right]\sin\left[\tfrac{1}{2}(A - B)\right]$$