Trent University: PHYS 4600Y – Quantum Physics: 2011-2012

Term Test 1: Tuesday Nov 1, 2011
Allowed: Formula sheets (given), calculator, 1 hour 50 minutes
Answer <u>all</u> questions; all questions worth equal marks

1 Sketch the probability density for the state function

$$\Psi(x,t) = Ae^{-(x/a)^2} e^{-i\omega t} \sin(kx)$$

and find the expectation value of momentum for a particle in this state.

2. For the energy eigenstates, $\phi_n(x) = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}$, of a particle in a 1-D infinite well with walls at $x = 0$ and $x = a$, show that $P(x) \equiv |\phi_n(x)|^2$ has maxima at values of x given by:

$$x_j = \frac{2j+1}{2n}a \quad \text{where } j = 0,1,2,....,n-1$$

3. Show that when a state function, $|\psi\rangle$, is expanded using a *discrete* basis set $\{|\phi_n\rangle\}$, then the expansion coefficients are given by $a_n = \langle\phi_n|\psi\rangle$. Similarly, show that when a state function, $|\psi\rangle$, is expanded using a *continuous* basis set $\{|\phi_k\rangle\}$, then the expansion coefficients are given by $b(k) = \langle\phi_k|\psi\rangle$.

4. If $A_{nl} \equiv \langle\phi_n|\hat{A}\phi_l\rangle$, show that $(\hat{A}^+)_{ln} = (A_{nl})^*$, where $\hat{A}^+$ denotes the Hermitian adjoint of $\hat{A}$.

5. Suppose a measurement of linear momentum is made on particles that are each described by a'top-hat' state function:

$$\Psi(x,0) = \sqrt{\frac{1}{a}} \qquad |x| < \frac{a}{2}$$

$$\Psi(x,0) = 0 \qquad |x| > \frac{a}{2}$$

What is the probability that a range of momentum from $-\pi\hbar/a \to \pi\hbar/a$ is measured?

[You may find one of the following integrals useful:

$\int_{-\pi}^{\pi} \frac{\sin^2 x}{x^2} dx = 0.903\pi$ $\qquad$ $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{x^2} dx = 0.774\pi$]

6. Consider a particle in a 1-D infinite well with walls at $x = 0$ and $x = a$, initially in a state given by

$$\Psi(x,0) = \sqrt{\frac{2}{5a}}(\sin(2\pi x/a) + 2\sin(\pi x/a))$$

Determine the state function at some time $t > 0$.