

Test I: Tuesday November 14, 2017

Answer question 1 **and** two out of the three remaining questions.

Each question carries equal marks. Show all working.

Allowed: 1 hour 50 mins. Calculator, formula sheets (given)

1a) A vector field $\vec{P}(\vec{r})$ has units of C/m^2 . What are the units of the expression $\int \vec{\nabla} \times \vec{P} \cdot d\vec{a}$?

b) A vector field is described by $\vec{v}(\vec{r}) = r^2(2 \cos \phi - 1)\hat{r} + 4r^2 \cos \theta \hat{\theta} + \sqrt{3}r^2(\sin \theta - \frac{1}{2})\hat{\phi}$ and we are interested in position P located at $(r, \theta, \phi) = (2, 30^\circ, 60^\circ)$. What are the (x, y, z) components of P, and what is the value of \vec{v} at P? Provide a 3D sketch showing point P and \vec{v} at P.

c) The electric field within a region of space is given in spherical coordinates by:

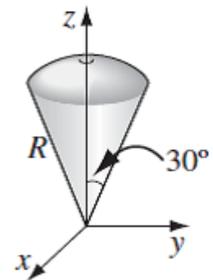
$$\vec{E}(\vec{r}) = \frac{k}{r} (3\hat{r} + 2 \sin \theta \sin \phi \hat{\theta} + \sin \theta \cos \phi \hat{\phi}), \quad \text{where } k \text{ is a constant}$$

Find the volume charge density within this region.

d) A uniformly-charged solid sphere has radius R and total charge Q . Find the electric field throughout all space due to this sphere. Sketch the magnitude of this field as a function of distance from the centre of the sphere.

2. A vector field is described by $\vec{v}(\vec{r}) = r^2 \hat{r} + 8r^2 \cos \theta \hat{\theta} + r^2 \hat{\phi}$.

Find the total flux of $\vec{v}(\vec{r})$ out from **both** the side of the cone shown, **and** from the top *spherical* surface of this cone.



3. Suppose a point charge, $+q$, is located at position $(0,0, z_0)$. Calculate the average electric potential taken over the surface of a sphere of radius R centred at the origin where $R < z_0$. Explain how this calculation relates to the *relaxation method*.

4. Find the energy of the uniformly-charged sphere described in Question 1d) above. [Two different formula are commonly used to calculate this, and you may (optionally) wish to check your work by arriving at a consistent answer from these two approaches.]