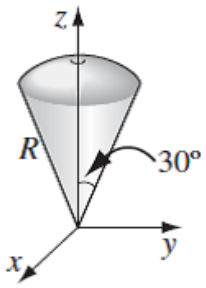
Trent University: PHYS 3200Y – Electricity and Magnetism 2016 – 2017

Test I: Wednesday November 16, 2016

Answer question 1 **and** two out of the three remaining questions.
Each question carries equal marks. Show all working.
Allowed: 1 hour 50 mins. Calculator, formula sheets (given)

1a) A vector field is described by $\vec{v}(\vec{r}) = r^2\,\hat{r} + 4r^2\cos\theta\,\hat{\theta} + r^2\,\hat{\phi}$. Find the flux of $\vec{v}(\vec{r})$ out from the side of the cone of length *R*, as shown (not the top surface).



b) A uniformly-charged cuboid has one vertex located at the origin, and extends along the *x*-axis to $x = 22\,\text{m}$, the *y*-axis to $y = 200\,\text{m}$, and the *z*-axis to $z = 2\,\text{m}$. The *volume charge density* is $6\,\text{nC/m}^3$. As one moves further away from this object it can often be viewed as a *sheet* of charge with *surface charge density*, σ, then at greater distances away as a *line* of charge with *linear charge density*, λ, then at greater distances away a *point* charge *Q*. Find the respective values of σ, λ and *Q*.

c) The electric field within a region of space is given in spherical coordinates by:

$$\vec{E}(\vec{r}) = \frac{k}{r}\left(3\hat{r} + 2\sin\theta\sin\phi\;\hat{\theta} + \sin\theta\cos\phi\;\hat{\phi}\right), \qquad \text{where } k \text{ is a constant}$$

Find the volume charge density within this region.

d) A particular charge configuration comprises four equivalent point charges, +*q*, located at the corners of a square of side *a*. Find the potential energy stored in this configuration.

2. Use Gauss' law to calculate the capacitance of the following two-conductor devices:
a) two large, flat, horizontal, conducting sheets of area *A*, separated by a small vertical distance *d*.
b) two thin, concentric, conducting spherical shells with radii *a* and *b* ($b > a$).

3. A uniformly charged circular loop of wire of radius *R* lies in the *xy*-plane, and carries linear charge density λ.
a) What do you expect the electric field to be at a point located a very large distance *z* above the centre of the loop, where $z >> R$? Give an equation for the field and explain your reasoning.
b) What do you expect the electric field to be at centre of the loop? Explain your reasoning.
c) Calculate the electric field at a general point a distance *z* above the centre of the loop, and confirm that your answers from (a) and (b) above are consistent with this.

4. a) Three thin, concentric, conducting spherical shells of increasing radii, centred at the origin, carry charges of $-1\,\text{mC}$, $+2\,\text{mC}$ and $-3\,\text{mC}$. The radius of the outer shell is 25 cm. What is the electric field just outside the outer shell?
b) A sheet of charge lies in the x-y plane and has uniform surface charge density $\sigma = 2\ \text{nC/m}^2$. There are other charged objects nearby. The electric field at a point close to the sheet and just above it is given by $\vec{E}(\vec{r}) = 30\hat{x} + 30\hat{y} + 30\hat{z}\ \text{V/m}$. Find the electric field at a point just the other side of the sheet.