Midterm: Wednesday November 18, 2015

Answer question 1 **and** two out of the three remaining questions.
Each question carries equal marks. Show all working.
Allowed: 1 hour 50 mins. Calculator, formula sheets (given)

1 a) Two vector fields are given by:

$$\vec{W}_1(\vec{r}) = y x^2 \hat{x} + x y^2 \hat{y} + 0\hat{z}, \qquad \text{and} \qquad \vec{W}_2(\vec{r}) = 2xy\,\hat{x} + x^2 \hat{y} + 0\hat{z}$$

Which one of these represents a possible *electrostatic* field, and why? For this field find the potential difference between points (1,1,1) and (2,1,1), assuming all values are in SI units.

b) The electric field within a region of space is given in spherical coordinates by:

$$\vec{E}(\vec{r}) = \frac{k}{r}\left(3\hat{r} + 2\sin\theta\sin\phi\,\hat{\theta} + \sin\theta\cos\phi\,\hat{\phi}\right), \qquad \text{where } k \text{ is a constant}$$

Find the charge density within this region.

c) Throughout the *xy*-plane an electric field has a constant direction but varying magnitude:

$$\left|\vec{E}(\vec{r})\right| = \frac{C}{r}, \quad \text{where } C \text{ is a constant}$$

where *r* is the distance from the origin. Within this plane $\vec{E}$ has only $\hat{y}$ and $\hat{z}$ components, and makes a constant angle of $30°$ with $\hat{z}$. Sketch the electric field lines for this field in this plane, and calculate its flux through a circle of radius *R* in the *xy*-plane, centred at the origin.

2. Three well-separated, non-conducting, droplets each have a radius of $250\,\mu\text{m}$ and contain a total charge uniformly-distributed within them of $+5\,\text{pC}$. The droplets come together and coalesce into a single spherical droplet with the same mass density as the original droplets, and with uniform charge density.

a) Find the magnitude of the electric field at the surface of each droplet before and after coalescence.
b) Find the electric potential, relative to infinity, at the surface of each droplet before and after coalescence.
c) Calculate the electric potential, relative to infinity, at the *centre* of each droplet before and after coalescence.

3. Consider two differently-shaped conductors: a solid sphere, and a solid egg-shaped structure.

a) Given that dry air breaks down when the electric field exceeds 2.5×10^6 V/m, what is the maximum total charge that can be placed on the sphere if it has radius (i) 10 cm, and (ii) 20 cm?
b) Assume the egg-shaped structure carries a net *negative* charge. Carefully sketch with an explanation the electric field lines, if any, (i) inside the conductor, (ii) slightly above the surface, and (iii) far from the surface.

4. A non-conducting sphere of radius *R* carries a charge density given by:

$$\rho(\vec{r}) = kr, \quad \text{where } k \text{ is a constant}$$

Find the total electrostatic energy of this charge configuration.