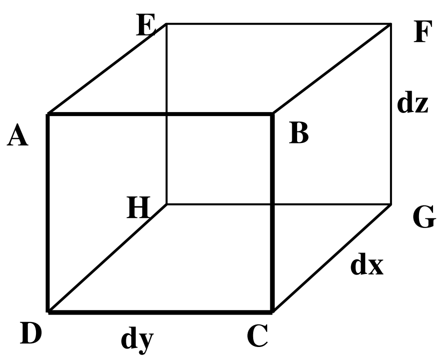
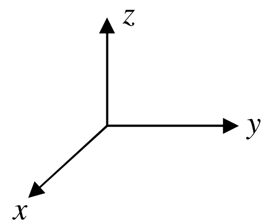
Midterm: Thursday November 14, 2013

Answer question 1 **and** two out of the three remaining questions.
Each question carries equal marks. Show all working.
Allowed: 1 hour 50 mins. Calculator, formula sheets (given)

1a) Consider the small volume $d\tau$ and axes shown:





Write down expressions for

i) $d\vec{l}$ from D to C
ii) $d\vec{l}$ from A to D
iii) $d\vec{l}$ from F to G
iv) $d\vec{a}$ for face ABCD
v) $d\vec{a}$ for face AEDH
vi) $d\vec{a}$ for face CDGH

b) A uniform electric field is given by

$$\vec{E}(\vec{r}) = \hat{x} + 2\hat{y} + 3\hat{z} \text{ V/m.}$$

i) Find the potential difference between the points (0,0,0) and (1,0,0).
ii) Find the potential difference between the points (1,0,0) and (1,1,0).

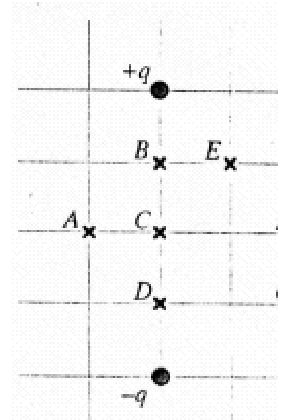
c) Consider first a *physical dipole*, composed of two charges $\pm Q$ separated in space by distance *a*.

i) Sketch physically realistic lines of electric field and of equal electrical potential (equipotential lines) due to this physical dipole.

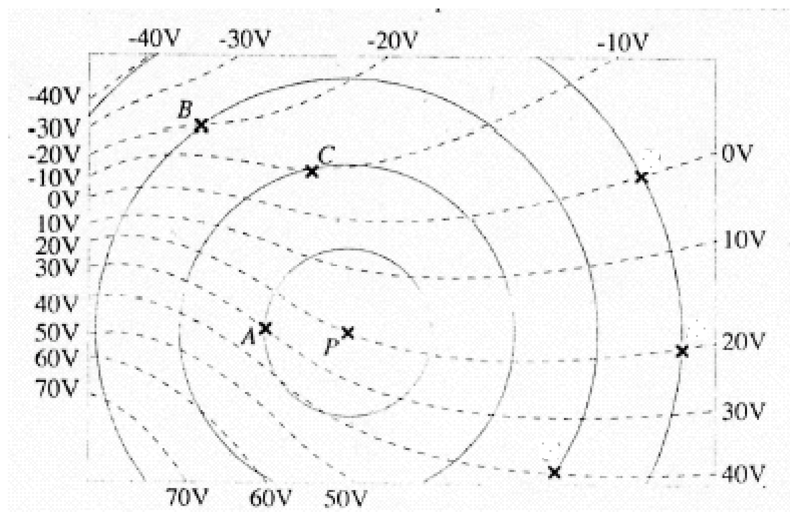
Suppose now a third charge +*q* is placed a distance *a* from each of the two charges, such that all three charges form an equilateral triangle.

ii) Use Gauss' law to find the total flux of the electric field through a closed Gaussian surface enclosing **only** the dipole.
iii) With the aid of a diagram denote the direction of the force experienced by the +*q* charge.

2.a) Two equal and opposite charges, $+q$ and $-q$, are fixed in space at the locations shown on the right, with each grid representing a square of side a. Five points in the vicinity of these charges are labeled A-E. Find, and simplify when appropriate, the electric potential at each of the labeled points relative to infinity.



b) The dashed lines in the figure below represent equipotentials with magnitudes labeled in volts in a region of space where there is an electric field. Also shown are circles centered at P with radius r_o , $2r_o$, $3r_o$ etc. Three points on these circles are labeled A to C. A positive charge, q, initially at rest is moved from P to each of these points in turn and ends at rest at these points. Find the work that must be done to move the charge q from P to A, from P to B, and finally from P to C.



c) Two charges each of +1 mC are situated at (-1,0,0) and (1,0,0). Find the electric field (it's a vector!) they produce at location (0,1,0). Find the force on a charge of +2 nC situated at this location. All coordinates are given in meters.

3. A large (approximated as infinite) plane is given by equation $x + y + z = 3$, where all coordinates are in meters. The plane has a uniform surface charge density $\sigma = 2\ \text{mC/m}^2$.

a) Sketch the plane $x + y + z = 3$.

b) Calculate the electric field (recall it's a vector) due to the plane of charge at the coordinate (7,8,9)

c) Calculate the force (it's a vector!) experienced by a charge of +4 nC situated at coordinate (7,8,9)

4. A thick spherical hollow shell with inner radius a, and outer radius b, carries a constant charge density:

$$\rho(\vec{r}) = k \qquad\qquad a \le r \le b$$

a) Find the electric field in the three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$.

b) Sketch $\left|\vec{E}\right|$ as a function of r, for the case $b = 2a$.

c) Substitute your solution from (a)(ii) into Gauss's law in differential form to confirm it is correct.