**Physics 321 – Electrictity and Magnetism. Midterm: Monday 21st November 2005**

**Answer <u>all</u> four questions (each carries equal marks). Show your working.**

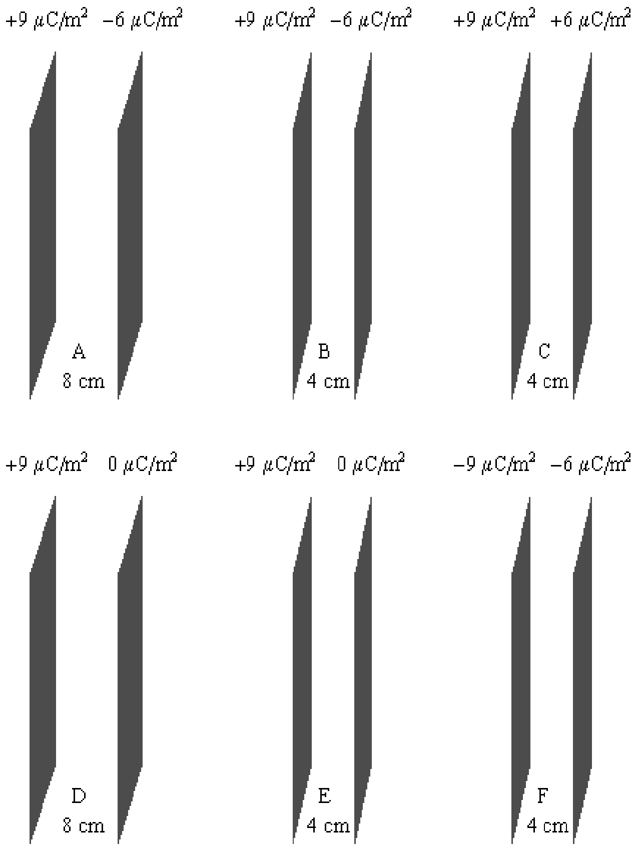
**Allowed: 2 hours. Calculator, 2-sided Formula Sheet, and up to 1 side of 8 ½×11” paper (written by you).**

1) With the aid of a diagram write down the general formula for the electric field, $\vec{E}(\vec{r})$, for cases (a) to (c):

a) due to a point charge, $q$, at some location in space
b) due to a configuration of point charges, $q_1$, $q_2$… $q_n$, at various locations
c) due to a continuous line charge distribution, $\lambda(\vec{r}')$.
d) What is the electric field at position $\vec{r} = 2\hat{x} + 4\hat{y} + 6\hat{z}$ due to a charge $+q$ at $\vec{r}_1' = \hat{x} + 2\hat{y} + 3\hat{z}$ and a charge $-q$ at $\vec{r}_2' = -\hat{x} + 3\hat{y} + 4\hat{z}$ ?
e) What is the electric field at distance $z$ above the center of a circular loop of radius $R$ in the $xy$ plane which carries a uniform line charge $\lambda_0$?

2) Write down the two forms (differential and integral) of Gauss’ law. Explain all terms and use one of these forms to calculate the electric field due to a single (assumed infinite) sheet of charge, with constant charge density $\sigma$. By considering two of these planes, one with $\sigma$, the other with $-\sigma$, parallel to the $xy$ plane determine the electric field inside a parallel plate capacitor.

Each diagram below shows two very large parallel insulating sheets (only a small portion near the centre of the sheets is shown; the distance between the sheets is very small compared to the dimensions of the sheet). The sheets are uniformly charged with charge densities and separations as shown. Find the ratios of the potential difference ($V_{left}$-$V_{right}$) between the sheets labeled $A$ to $F$.



3) With the aid of a diagram write down the general formula for the scalar potential, $V(\vec{\boldsymbol{r}})$, for cases (a) to (c) [assume that $\boldsymbol{V}(\infty) = 0$]:

a) due to a point charge, $q$, at some location in space
b) due to a configuration of point charges, $q_1$, $q_2$… $q_n$, at various locations
c) due to a continuous line charge distribution, $\lambda(\vec{\boldsymbol{r}}')$.
d) What is the scalar potential at position $\vec{\boldsymbol{r}} = 2\hat{\boldsymbol{x}} + 4\hat{\boldsymbol{y}} + 6\hat{\boldsymbol{z}}$ due to a charge $+2q$ at $\vec{\boldsymbol{r}}_1' = \hat{\boldsymbol{x}} + 2\hat{\boldsymbol{y}} + 4\hat{\boldsymbol{z}}$ and a charge $-q$ at $\vec{\boldsymbol{r}}_2' = 6\hat{\boldsymbol{y}} - 7\hat{\boldsymbol{z}}$ ? Is this positive or negative, and can you relate this to the work done to bring a test charge to $\vec{\boldsymbol{r}}$ ?
e) What is the scalar potential at distance $z$ above the center of a circular loop of radius $r$ in the $xy$ plane which carries a uniform line charge $\lambda_0$?

4. a) Write down Laplace's equation and Poisson's equation, explaining each term. Under what conditions would you use each of these?

b) Which of the following scalar functions satisfies Laplace's equation over all space?

i) $V(\vec{\boldsymbol{r}}) = 2 - 3x - 4y$
ii) $V(\vec{\boldsymbol{r}}) = 8xy - 4x^2 - 4y^2 - 10x + 8y + 12$
iii) $V(\vec{\boldsymbol{r}}) = 8xy - 4x^2 + 4y^2 - 10x + 8y + 12$

c) In a few sentences explain what is meant by the *method of images*. As an example, examine a charge $+q$ above a grounded conducting plane (in the $xy$ plane).

d) In a few sentences explain what is meant by the *method of separation of variables* to solve Laplace's equation. Assuming a Cartesian coordinate scheme, outline the steps taken, and explain the physics behind this method.