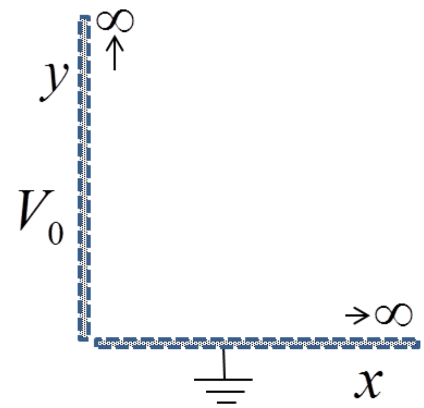
Term Test II: Tuesday February 6, 2018

Answer <u>three out of the following four</u> questions.
Each question carries equal marks. Show all working.
Allowed: 1 hour 50 mins. Calculator, formula sheets (given)

1. Two semi-infinite conducting sheets lie at $90°$ to each other. One sheet is grounded and lies in the *xz*-plane, of infinite extent along $\pm z$, and extending infinitely along the *positive* *x*-axis. The other sheet at potential V_0 lies in the *yz*-plane, of infinite extent along $\pm z$, and extending infinitely along the *positive* *y*-axis. At the origin there is a very small gap between the plates which we neglect in the following.



a) At first sight several possibilities seem viable for the potential field $V(\vec{r})$ in the space $x, y > 0$. These include the two possibilities:

$$V_1(s,\phi,z) = V_0 \sin\phi \quad \text{and} \quad V_2(s,\phi,z) = 2V_0\phi/\pi$$

One of these is the actual potential field. Which one is it, and provide justification for why this is so.
b) Then find the electric field, $\vec{E}(\vec{r})$ in the space $x, y > 0$, in both cylindrical coordinates, and also in Cartesian coordinates.
c) Sketch some equipotentials and also the electric field for $x, y > 0$.
d) Suppose an electric dipole with dipole moment given in Cartesian coordinates by $\vec{p} = (p_x, 0)$ is now placed at location (x_0, y_0), where $x_0 > 0; y_0 > 0$. Assume this dipole is sufficiently weak that we can neglect any image charges. Find the force exerted on this dipole, and check it is physically consistent with what you might expect when $x_0 << y_0$, and when $y_0 << x_0$

2. A capacitor comprises two large rectangular metal plates of dimensions *a* and *b*, held parallel to each other and separated by a distance *d*. A dielectric slab with dielectric constant ε_r fills the gap between the plates. A battery of voltage V_0 is then attached to the capacitor. In the following, neglect any edge effects.
a) From the equations given on your formula sheet derive the capacitance of this device.

The dielectric slab is then partially pulled out of the plates along the *a* direction such that only length *x* (instead of *a*) remains between the plates.

b) For each of the two now-distinct regions of the capacitor find the free surface charge density on each plate. Also find the bound surface charge density of the slab.
c) Determine the new capacitance, and check it is consistent with what you found in part (a).

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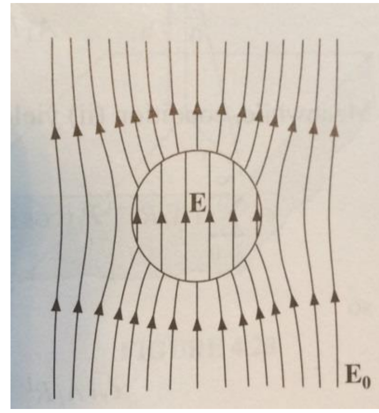
3. Consider a sphere with dielectric constant ε_r, of radius *R*, and no free charge, centred at the origin, and placed in a uniform electric field $\vec{E}_0$ parallel to the *z*-axis. [Therefore at large *r* the potential $\rightarrow -E_0 r\cos\theta$]

a) Which three boundary equations apply that provide information about $V_{in}(r,\theta)$ and/or $V_{out}(r,\theta)$ at various values of *r*?

b) Provide a general formula for $V_{in}(r,\theta)$.

c) Provide a general formua for $V_{out}(r,\theta)$.

d) By comparing coefficients from these expressions subject to the boundary conditions, find $V_{in}(r,\theta)$, and check it is consistent with what you would expect if there were no dielectric sphere.



4. a) A long, otherwise straight wire contains a ½ turn, flat, circular ring of radius *R*, as shown below. Current *I* flows from left to right. From the equations given on your formula sheet determine the magnetic field (magnitude and direction) at the centre of the ring. Assume $\hat{x}$ and $\hat{y}$ are in the directions shown.

b) A square loop of side *a* is shown below at a distance *a* from a long straight wire, carrying a current I_1 to the right. A clockwise current I_2 is created in the square. From the equations given on your formula sheet determine the total force (magnitude and direction) on the square. Again assume $\hat{x}$ and $\hat{y}$ are as shown.