Trent University: PHYS 3200Y – Electricity and Magnetism 2013 – 2014

Term Test II: Thursday February 6, 2014

Answer three out of the following four questions.
Each question carries equal marks. Show all working.
Allowed: 1 hour 50 mins. Calculator, formula sheets (given)

1a) Four charges are placed at the following (x,y,z) coordinates:

$3q$ at $(a,a,0)$, $-2q$ at $(0,-a,0)$, q at $(-a,-a,0)$ and $-2q$ at $(0,a,0)$

Find the electric dipole moment, $\vec{p}$, of this charge distribution and denote the direction $\hat{p}$ on a suitable plot.

b) A uniform 0.10 T magnetic field is parallel to the floor and points in the eastward direction. A straight segment of 1.0 mm diameter copper wire (mass density 8920 kg/m^3) is also parallel to the floor. What minimum current must flow through the wire, and in what direction must the current flow, in order to levitate the wire in the magnetic field?

2. A sphere of radius R, centred at the origin, carries a volume charge density:

$$\rho(\vec{r}) = k\frac{R}{r}(R-2r)\sin\theta$$

a) With the aid of a clear diagram depict the charge distribution within this sphere.
b) Explicitly expand the multipole formula for the long-range potential field due to a charge distribution located at the origin, up to and including the quadrupole term.
c) For the charge distribution given here find both the monopole and dipole contributions to the long-range potential along the z-axis.

3a) A large, charged slab of thickness $2a$ lies in the xy-plane, with its upper and lower edges at $z = \pm a$. The volume charge density inside the slab is given by

$$\rho(\vec{r}) = \rho_0\left(1-\frac{|z|}{a}\right).$$

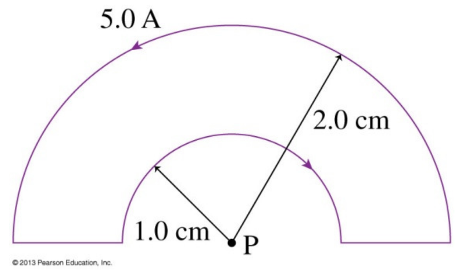
Sketch the charge distribution within the slab and find the electric field (its magnitude and direction) outside this slab of charge.

b) Suppose now two such equal and oppositely-charged slabs are placed parallel to each other with a gap of size b between them. Suppose further a slab of *uncharged* insulating material with dielectric constant ε_r and total thickness along z less than b is inserted into the gap. Using the electric displacement $\vec{D}$–field demonstrate that the magnitude of the electric field inside the insulating slab, $E_{dielectric}$, is reduced compared to the field in the gap outside the insulating slab, E_0, according to

$$E_{dielectric} = \frac{E_0}{\varepsilon_r}.$$

Also determine the bound surface charge density, σ_b, on each face of the uncharged insulating slab.

4.a) Starting with the Biot-Savart law find the magnitude of the magnetic field at the centre of a circular loop of wire of radius R carrying a current I. Then, by adapting this formula appropriately, find the magnitude and direction of the magnetic field at point P, shown to the right.



b) It can be shown that the magnitude of the magnetic field $\vec{B}(\vec{r})$ at a perpendicular distance s from a finite length of wire carrying current I and with ends that are located at angles θ_1 and θ_2 from the perpendicular to the wire is given by

$$B = \frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right).$$

By applying this formula to an infinite wire, find the magnitude of the magnetic fields at points a, b, and c shown to the right.