

Electricity and Magnetism Midterm: Wednesday 28th February 2008

Answer five out of the six questions (each carries equal marks). Show your working.

Allowed: 2 hours. Calculator, 4-sided Formula Sheet (given)

- 1) a) Use Gauss's law to derive an expression for the electric field, \vec{E} , at an arbitrary position outside a long straight cylinder of radius R , with uniform charge per unit length λ .
 b) Electric field breakdown (i.e. arcing or sparking) occurs in damp air when $|\vec{E}|$ exceeds approximately 10^5 V/m. If the radius of the hydro line in part (a) is 0.5 cm, find the maximum allowed charge per unit length on the wire.

2. An amount of charge q is uniformly spread on the surface of an insulating disk of radius a . Find the potential V , at any point on the axis of rotational symmetry. From this, or otherwise, find the electric field at this point.

3. A thin metal shell of radius b has charge Q .
 a) What is its capacitance?
 b) What is the energy density of the electric field at distance r from the sphere's centre?
 c) What is the total energy in the field?

4. An ideal electric dipole with dipole moment \vec{p}_1 is located at the origin, with \vec{p}_1 parallel to \hat{z} . A second dipole with moment \vec{p}_2 is located at $(0,0,z)$, with \vec{p}_2 also parallel to \hat{z} .
 a) From the potential in the region of \vec{p}_2 , find the electric field at \vec{p}_2 .
 b) Find the force on \vec{p}_2 .

5. The space between the square plates (of sides L) of a large parallel-plate capacitor, which are separated by $3a$, is filled with three slabs of linear dielectric material. Each slab has thickness a , and slab 1 has relative permittivity (dielectric constant) of 2, slab 2 has relative permittivity of 3, and slab 3 has a relative permittivity of 4. The free charge density on the top plate is σ , and on the bottom plate $-\sigma$.
 a) Find the electric displacement \vec{D} in each slab.
 b) Find the potential difference between the plates.
 c) Find the total energy stored in the system (i.e. energy in the field and the dielectric)
 d) Compare your answers for (b) and (c) to an identical case where there is no dielectric between the plates.

6. a) A long, otherwise straight wire contains a $\frac{1}{2}$ turn, flat, circular ring of radius R , as shown in Figure 1a below. A current flows from left to right. What is the magnetic field at the centre of the ring?
 b) A square loop of side a is shown in Figure 1b below at a distance a from a long straight wire, carrying a current I_1 to the right. If a clockwise current I_2 is created in the square, what is the force (magnitude and direction, of course) on the square?



Figure 1a

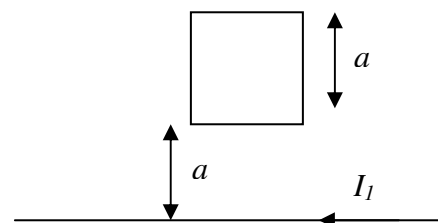


Figure 1b