Physics 321 – Electrictity and Magnetism. Midterm: Wednesday 15th February 2006

Answer all four questions (each carries equal marks). Show your working.

Allowed: 2 hours. Calculator, 2-sided Formula Sheet,
Up to 1 side of 8 ½×11" paper (equations, no text).

1) a) Given a hollow charged sphere with known surface charge $\sigma(\theta)$ placed at the origin, we know from the separation of variables technique for solving Laplace's equation that the general form of the scalar potential is given by:

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos(\theta))$$

What can you immediately say (and why?) about the A_l and B_l coefficients both inside the sphere and outside the sphere? (Do not solve this problem completely).

b) What is the formula for the electric dipole moment, $\vec{p}$, due to:
i) a configuration of point charges, q_1, $q_2 \ldots q_n$, at various locations in space
ii) a continuous charge distribution, $\rho(\vec{r}')$, and
iii) what dependence does the scalar potential due to an electric dipole have on r, the distance from the dipole?
iv) what dependence does the electric field due to an electric dipole have on r, the distance from the dipole?

c) What is the definition of atomic polarizability? (Explain all terms used)

d) What is the formula for the torque on an electric dipole, $\vec{p}$, in an electric field, $\vec{E}$?

e) What is the formula for the force on an electric dipole, $\vec{p}$, in an electric field, $\vec{E}$?

2) Four charges are placed at the following (x,y,z) coordinates:

$3q$ at $(a, a, 0)$, $-2q$ at $(0,-a,0)$, q at $(-a,-a,0)$ and $-2q$ at $(0,a,0)$.

a) Find the electric dipole moment, $\vec{p}$, of this charge distribution.
b) Find the electric potential $V(\vec{r})$ at large r due to this distribution in spherical polar coordinates
c) Find the electric field $\vec{E}(\vec{r})$ at large r in spherical polar coordinates
d) If $q = 1.6\times10^{-19}$ C and $a = 0.1$ nm, what is the largest electric field strength at $r = 1$ μm?

3) A dielectric cube, with side *a*, centred at the origin, carries a "frozen-in" polarization $\vec{P}(\vec{r}) = k\vec{r}$, where *k* is a constant.

a) Find all charge and its location for this system

b) Where did this charge come from?

c) What is the total charge of this cube?

4) With the aid of a clear diagram in each case write down the contribution to the magnetic field, $\vec{B}(\vec{r})$, for cases (a) to (c):

a) due to a small length, $d\vec{l}\,(\vec{r}\,')$, of steady current, *I*

b) due to a small area of steady surface current density, $\vec{K}(\vec{r}\,')$

c) due to a small volume of steady volume current density, $\vec{J}(\vec{r}\,')$

Suppose there are two infinite (i.e. very long) parallel wires both moving at speed *v* along their longitudinal direction. The linear charge density of each wire is λ. What is the electric force on each wire, and in which direction does it point? What is the magnetic force on each wire, and in which direction does it point? At what speed *v* are these equal?