Trent University: PHYS 3200Y – Electricity and Magnetism 2017 – 2018

Final Exam: Wednesday April 17, 2018

Answer **five** questions from part A (60% total) and **two** questions from part B (40% total)

Allowed: 3 hours, calculator, formula sheets (given)

For **each** question arrive at your solution using formulae given on the formula sheets, and identify which formulae you use.

Part A

A1. A long, non-conducting cylinder lying along the *z*-axis has radius *R* and a positive volume charge density given by:

$$\rho(s,\phi,z) = \rho_0\left(1 - \frac{s}{R}\right).$$

a) Sketch how this charge density depends on *s* at a given ϕ and *z*.

b) Find the electric field, $\vec{E}(s,\phi,z)$, inside the cylinder.

A2. A spherical liquid droplet of radius 250 μm has charge of 50 pC uniformly distributed over its surface.

a) Assigning the electric potential far away from the drop to be zero volts, what is the potential at the surface of the droplet?

b) If three of these droplets coalesce together and form a single sphere with same mass density, and the total charge uniformly spread over its surface, what is the electric potential at the surface of this sphere?

A3. a) Use Maxwell's equations and the definition of the vector potential to deduce the following equation from the formula sheet for static fields:

$$\nabla^2 \vec{A}(\vec{r}) = -\mu_0 \vec{J}(\vec{r}) \quad ,$$

and state any assumptions you make.

b) For an *ohmic conductor* write down how $\vec{J}$ depends on $\vec{E}$.

c) In a *superconductor* the analogous relationship is $\vec{J}(\vec{r}) = K\,\vec{A}(\vec{r})$. Given this, demonstrate that if the entire semi-infinite region of space $z > 0$ is occupied by a superconductor, and if $\vec{A}(x,y,z) = A_x\,\hat{x}$ only, then the vector potential inside the superconductor decays from its value at $z = 0$ according to $A_x = A_0 e^{-z/\lambda}$. Find an expression for λ in terms of K.

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A4. It can be concluded from Maxwell's equations that for a single harmonic plane wave the following relationship holds between the real, physical, electric and magnetic fields:

$$\vec{B}(\vec{r},t) = \frac{\vec{k}\times\vec{E}(\vec{r},t)}{\omega} \quad ,$$

where the variables have their usual meanings. Suppose the electric field is described by the complex wave $\tilde{\vec{E}}(\vec{r},t) = E_0 e^{i(k_0 z-\omega t)}\hat{y}$ where E_0 is a value with no imaginary component.

a) What is the wave's real electric field?
b) What is the wave's real magnetic field?
c) If the electric field is instead described by $\tilde{\vec{E}}(\vec{r},t) = E_0 e^{i(k_0 z-\omega t+\pi/2)}\hat{x} + E_0 e^{i(k_0 z-\omega t)}\hat{y}$, then describe with brief justification the *polarization* of this wave.

A5. A spherical conductor of radius *a* carries a charge *Q*. It is surrounded by a uniform linear dielectric material of susceptibility χ_e out to radius *b*. Find the electric field, $\vec{E}(\vec{r})$, and the displacement field, $\vec{D}(\vec{r})$, for all points in space.

A6. A wire loop of some arbitrary shape with area 0.65 m^2 and resistance 10 Ω lies in the *xy*-plane. A magnetic field exists in this region of space given by:

$$\vec{B}(\vec{r},t) = 0.05\cos\left(10^3 t\right)\left(\frac{\hat{y}+\hat{z}}{\sqrt{2}}\right).$$

a) Find the magnitude of the induced current through the loop.
b) Place on top of each other two graphs: one of $B_z(t)$, and one of the *clockwise* induced current as seen looking *down* the z-axis towards the xy plane.

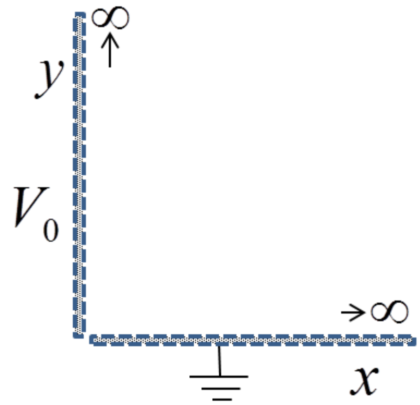
Part B

B1. A hollow solenoid lying along the *z*-axis has the following properties: length = 2 m; radius = 0.1 m; total number of turns = 2000; resistance = 0.1 Ω.

a) Derive the expression for the magnetic field within the solenoid, ignoring end effects. What value does it take when *I* = 2000 A?
b) Including end effects, what is the approximate magnetic field near the ends of the solenoid?
c) Calculate the self-inductance of the solenoid.
d) What is the stored energy when the solenoid is operated with this current?
e) Suppose this solenoid is connected to a switch and a 20 V DC supply. Find and sketch the resulting current through the solenoid as a function of time after closing the switch, and determine the *time constant* (in seconds) of the circuit.

[Note that a differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$ can be solved by first multiplying each side by $e^{\int P(x)dx}$, and then the left hand side can be represented as a derivative of a single term].

B2. Two semi-infinite conducting sheets lie at $90°$ to each other. One sheet is grounded and lies in the *xz*-plane, of infinite extent along $\pm z$, and extending infinitely along the *positive* *x*-axis. The other sheet at potential V_0 lies in the *yz*-plane, of infinite extent along $\pm z$, and extending infinitely along the *positive* *y*-axis. At the origin there is a very small gap between the plates which we neglect in the following.



a) At first sight several possibilities seem viable for the potential field $V(\vec{r})$ in the space $x, y > 0$. These include the two possibilities:

$$V_1(s,\phi,z) = V_0 \sin\phi \quad \text{and} \quad V_2(s,\phi,z) = 2V_0\phi/\pi$$

One of these is the actual potential field. Which one is it, and provide justification for why this is so.

b) Then find the electric field, $\vec{E}(\vec{r})$ in the space $x, y > 0$, in both cylindrical coordinates, and also in Cartesian coordinates.

c) Sketch some equipotentials and also the electric field for $x, y > 0$.

d) Suppose an electric dipole with dipole moment in Cartesian *xy*-coordinates of $\vec{p} = (p_x, 0)$ is now placed at location (x_0, y_0), where $x_0 > 0; y_0 > 0$. Assume this dipole is sufficiently weak that we can neglect any image charges. Find the force exerted on this dipole, and check it is physically consistent with what you might expect when $x_0 << y_0$, and when $y_0 << x_0$.

B3. One material (labelled 1) with $\mu_{r1} = 15$ lies below a second material with $\mu_{r2} = 1$, with the interface between the two comprising the *xy*-plane. The magnetic field in material 1 near the interface is $\vec{B}_1 \equiv (B_{1x}, B_{1y}, B_{1z}) = (1.2, 0.8, 0.4)$ T. No free currents flow.

a) Find $\vec{H}_1$ (the $\vec{H}$ - field in material 1), near the *xy*-plane, with the correct units.

b) By considering the boundary conditions for $\vec{H}$ at the interface between two materials, find $\vec{H}_2$ (the $\vec{H}$ - field in material 2) in terms of the unknown value H_{2z}.

c) With a similar approach find $\vec{B}_2$ (the $\vec{B}$ - field in material 2) in terms of values B_{2x} and B_{2y}.

d) Then find H_{2z}, B_{2x} and B_{2y}.

e) Verify that the angle between $\vec{B}_1$ and $\vec{B}_2$ is $61°$.