Final Exam: Saturday April 16, 2016

Answer **five** questions from part A (60% total) and **two** questions from part B (40% total)

Allowed: 3 hours, calculator, formula sheets (given)

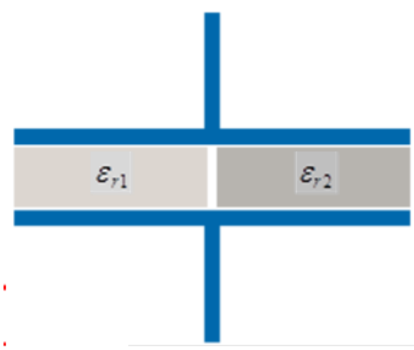
For **each** question arrive at your solution using formulae given on the formula sheets, and identify which formulae you use.

Part A

A1. Suppose $V(\vec{r}) = xy^2z^2$ volts in a region of space defined by $0 < x, y, z < 2$.
(a) Find the charge density $\rho(\vec{r})$ in this region, and also the total charge in this cube.
(b) If this charge density is traveling with a velocity of $100\,\hat{z}$ m/s, find the instantaneous current crossing the surface defined by $z = 1 \quad ; \quad 0 < x, y < 2$.

A2. Consider 3 straight, long, parallel wires in the *xz* plane, each with current *I* flowing along $+\hat{\mathbf{z}}$, and passing through $x = -d;\ x = 0;\ x = +d$ respectively. Assume the wires are thin compared to *d*.
a) Sketch the **B**-field in the xy-plane
b) Find the two locations on the *x*-axis between the three wires where the magnetic field is zero (note that these are <u>not</u> $x = \pm d/2$).

A3. A parallel-plate capacitor consists of two horizontal plates, each of area *A* and separation *d*. Two different dielectrics ($\varepsilon_{r1}, \varepsilon_{r2}$) are inserted as shown in the figure, each filling half the space.
a) If the potential difference between the plates is V_0, find the surface charge density on the left side of the plates, $\pm\sigma_1$, and on the right side, $\pm\sigma_2$, neglecting edge effects.[Hint: using the $\vec{D}$-field may help here].
b) Find the capacitance of the capacitor with the dielectrics in place.



A4. It can be shown that the magnetic field in the gap between the two conductors of a cable composed of a pair of long, coaxial, hollow conductors, each carrying current *I* in opposite directions is given by:

$$\vec{B}(s, \phi, z) = \frac{\mu_0 I}{2\pi s}\hat{\phi} \ ,$$

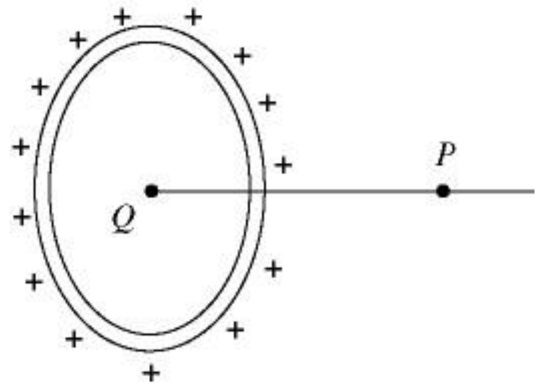
where the cable is assumed to lie along the z-axis. If the radius of the inner conductor is *a* and that of the outer conductor is *b*, find the self-inductance per unit length of this cable.

A5. Circular plates of a capacitor are 5.0 cm in radius, 2.0 mm apart, and have air between them. The voltage across the plates changes at a rate of 60 V/s. Determine:
a) the rate of change of the electric field between the plates
b) the displacement current, I_d , between the plates
c) the magnetic field inside the capacitor at a distance 2.5 cm from the central axis of the plates.

A6. Show that a uniform magnetic field, $\vec{B}_0$, can be represented by the vector potential $\vec{A}(\vec{r}) = \frac{1}{2}(\vec{B}_0 \times \vec{r})$

Part B

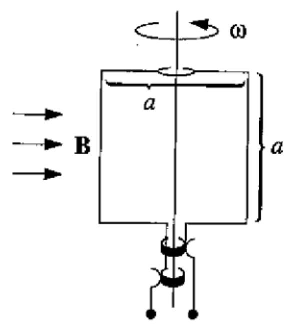
B1.a) In the figure to the right, a ring of radius R carries a charge of Q' uniformly distributed over it. A point charge Q is placed at the center of the ring. Find an expression for the value of the point charge Q required to make the electric field equal to zero at point P, which is on the axis of the ring, distance z from its center.



b) Find the value of Q if $R = 0.71\,\text{m}; Q' = 580\,\text{nC}$ and $z = 0.73\,\text{m}$. What value does Q tend towards as $R/z \to 0$?

B2. a) A metal wire of length 0.2 m lies initially along the x-axis, and has a velocity given (in SI units) by $\vec{v} = 2.5\sin(1000\,t)\,\hat{z}$. A magnetic field exists throughout the region given by $\vec{B} = 0.04\,\hat{y}\ \text{T}$. Find the emf induced by this moving wire, and also determine its polarity as a function of time.

b) A square coil of side a rotates about the z-axis at angular speed ω rad/s in a transverse magnetic field of $B\ \text{T}$, as shown. Find an expression for the emf induced by this ac generator.



B3. One material (labelled 1) with $\mu_{r1} = 15$ lies below a second material with $\mu_{r2} = 1$, with the interface between the two comprising the xy-plane. The magnetic field in material 1 near the interface is $\vec{B}_1 \equiv (B_{1x},\ B_{1y},\ B_{1z}) = (1.2, 0.8, 0.4)\ \text{T}$. No free currents flow.

i) Find $\vec{H}_1$ (the $\vec{H}$ - field in material 1), near the xy-plane.

ii) By using the boundary conditions for $\vec{H}$ at the interface between two materials, find $\vec{H}_2$ (the $\vec{H}$ - field in material 2) in terms of the unknown value H_{2z}.

iii) Use a similar approach to find $\vec{B}_2$ (the $\vec{B}$ - field in material 2) in terms of unknown values B_{2x} and B_{2y}.

iv) Then find H_{2z}, B_{2x} and B_{2y}.

v) Verify that the angle between $\vec{B}_1$ and $\vec{B}_2$ is $61°$.