Final Exam: Sunday April 13, 2014

Answer **six** questions from part A (60% total) and **two** questions from part B (40% total)

Allowed: 3 hours, calculator, formula sheets (given)

For **each** question arrive at your solution using formulae given on the formula sheets, and identify which formulae you use.

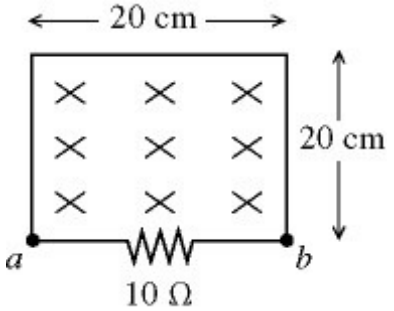
Part A

A1. Find the electric dipole moment of a cubical volume of charge that extends from $-a \le x, y, z \le a$ with $\rho(\vec{r}) = Ax$, where A is a constant.

A2. A proton is released from rest at the surface of a 1 cm radius metal sphere that has been charged to 100V.
a) What is the charge on the sphere?
b) What is the proton's speed at a distance 1 cm from the surface of the sphere?

A3. A long uncharged wire carries a 10 A current from left to right. An electron 1.0 cm above the wire travels to the right at a speed of 1.0×10^7 m/s. Find the magnitude and direction of the force on the electron due to the magnetic field.

A4. As shown in the figure to the right, a wire and a 10 Ω resistor are used to form a circuit in the shape of a square, 20 cm by 20 cm. A uniform but time-varying magnetic field is directed into the plane of the circuit. The magnitude of the magnetic field is decreased linearly from 1.50 T to 0.50 T in a time interval of 63 ms. Find the induced current and its direction through the resistor in this time interval.



A5. A long wire solenoid with 15 turns per centimeter carries a current of 1.00 A.
a) Find the magnetic field inside the solenoid.
b) Suppose the core is now filled with iron and the resulting magnetic field inside the solenoid is 2.46 T. Find the magnitude of the magnetization field, $\vec{M}$, inside the iron.

A6. Show that the magnetic field lines on either side of two linear magnetic materials, in the absence of any free current, obeys

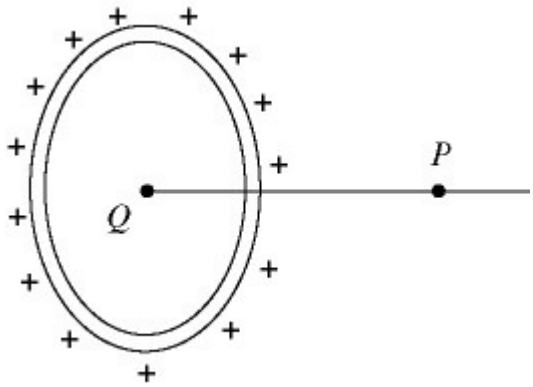
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$$

where θ_i is the angle between magnetic field $\vec{B}_i$ and the normal to the surface in the i-th material with magnetic permeability μ_i.

A7. Show that a uniform magnetic field, $\vec{B}_0$, can be represented by the vector potential $\vec{A}(\vec{r}) = \frac{1}{2}\left(\vec{B}_0 \times \vec{r}\right)$

Part B

B1.a) In the figure to the right, a ring of radius R carries a charge of Q' uniformly distributed over it. A point charge Q is placed at the center of the ring. Find the value of the point charge Q required to make the electric field equal to zero at point P, which is on the axis of the ring, distance z from its center.



b) Find the value of Q if $R = 0.71\,\text{m}; Q' = 580\,\text{nC}$ and $z = 0.73\,\text{m}$. What value does Q tend towards as $R/z \to 0$?

B2. Two 1.0 cm × 2.0 cm rectangular metal plates are held parallel to the floor, separated by a vertical distance of 1.0 mm.

a) What charge must be placed on each plate to create a uniform electric field of 2.0×10^6 N/C in the space between the plates?

b) Now assume that a dielectric slab with $\varepsilon_r = 3$ and the same dimensions as the plates but of thickness $d < 1.0\,\text{mm}$ is placed between the plates. Find the surface charge density induced on the upper and lower faces of this slab.

B3. a) The magnetic field inside a 5.0 cm diameter solenoid is 2.0 T and decreases at 3.10 T/s.

i) What is the magnitude of the electric field inside the solenoid on the axis?

ii) What is the magnitude of the electric field inside the solenoid at a point 1.60cm from the axis?

b) Two metal objects are embedded in weakly conducting material of conductivity σ, and separated from all other objects. Show that the resistance between them is related to the capacitance of the arrangement, C, by:

$$R = \frac{\varepsilon_0}{\sigma C}$$