Physics 321 – Electricity and Magnetism. Final Exam : Thursday 10th April 2008

Answer **four** questions from **part A** (total is worth 40 % of the final mark)

Answer **three** questions from **part B** (total is worth 60% of the final mark)

Allowed: 3 hours. Calculator, two sheets of 2-sided formulae (given),

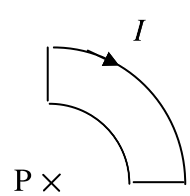
For each problem briefly explain the equations you choose to use – simply writing down a set of equations from the formula sheets will not earn any marks.

Part A:

1. Two positive charges (+8.0 mC and +2.0 mC) are separated by 300 m. A third charge is placed a distance *r* from the +8.0 mC charge in such a way that the resultant electric force on the third charge due to the other two charges is zero. Find the distance *r*.

2. An infinite line charge of charge density $\lambda = 0.3$ μC/m lies along the *z* axis and a point charge $q = 1.0$ μC is placed on the *y* axis at $y = 1.0$ m. Find the electric field, $\vec{E}$, at a point on the *x* axis where $x = \sqrt{3}$ m.

3. Find the magnetic field, $\vec{B}$ at point P for the steady current configuration shown below - comprising two $90°$ arcs of radius *a* and *b* with centres of curvature at point P ($a < b$):



4. A large parallel-plate capacitor connected to a battery stores twice as much energy with a particular linear dielectric as it does with air in the gap. Briefly explain in terms of all charges present in the system how this can be the case, and find the susceptibility, χ_e, of the dielectric.

5. Briefly explain using your own words what equation 3.1 on Formula Sheet 2 means, what the terms represent, and under what circumstances this formula can be applied.

6. Maxwell's equations are commonly written in two equivalent forms: either in the "*exclusively* $\vec{E}, \vec{B}$ *form*", or in the "*mixed* $\vec{E}, \vec{B}, \vec{D}, \vec{H}$ *form*". By deriving one of these forms from the other, show that these two are indeed equivalent to each other.

Part B:

1. Consider two concentric cylindrical metal plates. The inner plate with radius a is held at zero volts and the outer plate with radius b is held at voltage V_o.
(a) Write down Laplace's equation in a coordinate system suitable for this problem.
(b) Ignoring edge effects, find the electric potential in the region between the plates. What is the equation for this potential, V_{mid}, exactly mid-way between the plates?
(c) By considering the functional form of the electric field in the region between the plates, qualitatively explain the limiting behaviour of V_{mid} as $b >> a$, and as $b \to a$ (you may wish to evaluate V_{mid} for a range of b/a values, but note that in all cases $b > a$).

2. Suppose $V(\vec{r}) = xy^2z^2$ volts in a region of space defined by $0 < x, y, z < 2$.
(a) Find the charge density $\rho(\vec{r})$ in this region.
(b) If this charge is traveling with a velocity of $100\,\hat{z}$ m/s, find the instantaneous current crossing the surface defined by $0 < x, y < 2$; $z = 1$.

3. A circuit contains a resistance of 25 Ω, an inductance of 5.4 mH, a battery of emf = 9.0 V and a switch, all in series with each other.
(a) Find the equation for the current in the circuit, $I(t)$, from the time that the switch is first closed.
(b) What is the time constant for this circuit?
(c) After a time of 50 μs, what current is flowing in this circuit?
(d) Find the energy stored in the inductor when a steady current is achieved.
(You may recall that the solution, $y(x)$, to the differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are at most functions of x only, can be found by multiplying both sides of this equation by $e^{\int P dx}$).

4.(a) A long straight wire with radius 1 cm has an internal $\vec{H}$-field given in terms of cylindrical coordinates in S.I. units:

$$\vec{H}(s,\phi,z) = 4.77\times10^4\left(\frac{s}{2} - \frac{s^2}{0.03}\right)\hat{\phi} \quad \text{A/m}$$

What is the total free current in the wire?

(b) A magnetic field is described by the following vector field in cylindrical coordinates:

$$\vec{B}(s,\phi,z) = \frac{0.2}{s}(\sin\phi)^2\,\hat{z}$$

Find the total magnetic flux through the $z = 0$ plane for $s \le 0.05$ m.