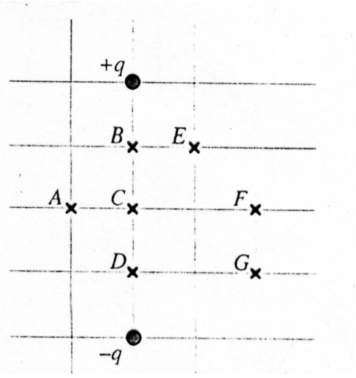
Physics 321 – Electricity and Magnetism. Final Exam : Wednesday 19th April 2006

Answer <u>four</u> out of the five questions (each carries equal marks).

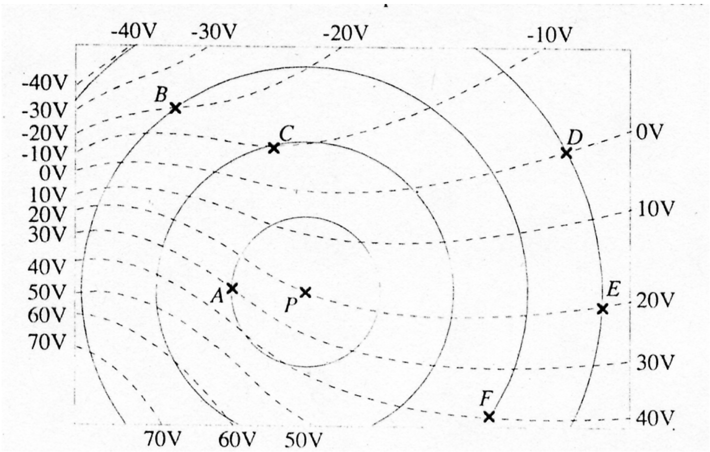
Allowed: 3 hours. Calculator, 2-sided formula sheet (given),
Up to 2 sides of 8 ½×11" paper (equations, no text).

For each problem <u>briefly explain</u> the equations you choose to use – simply writing down a set of equations from your formula sheet will not earn any marks.

1. a) Two equal and opposite charges are fixed in space at the locations shown below, with each grid representing a square of side a. Seven points in the vicinity of these charges are labeled A-G. Find the electric potential at each of the labeled points.



b) The dashed lines in the figure below represent equipotentials (with magnitudes labeled in the margins of the drawing) in a region where there is an electric field. Also shown are circles centred at P with radius r_0, $2r_0$, $3r_0$ etc. Six points on these circles are labeled A to F. A positive charge, q, initially at rest is moved from P to each of these points and ends at rest at these points. Find the work that must be done by an external agent to move the positive charge from P to each of the labeled points.



c) For the cubical volume $0 \le x, y, z \le a$, find the total charge and electric dipole moment (with respect to the origin) for the charge distribution:

$$\rho(x, y, z) = Axy$$

where A is a constant.

2. a) Two metal objects are embedded in weakly conducting material of conductivity σ, and separated from all other objects. Show that the resistance between them is related to the capacitance of the arrangement by:

$$R = \frac{\varepsilon_0}{\sigma C}$$

b) An electron is introduced into a region of uniform electric and magnetic fields such that $\mathbf{E}(\mathbf{r}) = E_0\hat{\mathbf{z}}$ and $\mathbf{B}(\mathbf{r}) = B_0\hat{\mathbf{y}}$. What must be the entering velocity (speed **and** direction) of the electron such that it continues unperturbed through this region?

c) Show that a uniform magnetic field, $\mathbf{B}_0$, can be represented by the vector potential $\mathbf{A} = \frac{1}{2}(\mathbf{B}_0 \times \mathbf{r})$

3) A solenoid is designed to generate a magnetic field over a large volume. Its dimensions are as follows: length = 2 m, radius = 0.1 m, total number of turns = 2000. (Edge effects should be neglected).
a) Use one or more of Maxwell's equations to derive the magnetic field within the solenoid. What value does it take when I = 2000 A?
b) Calculate the self-inductance, L, of the solenoid
c) What is the stored energy when the solenoid is operated with this current?
d) The total resistance of the solenoid is 0.1 Ω. Derive the equation describing the transient current as a function of time immediately after connecting this solenoid to a 20 V d.c. supply. Sketch this function. What is the time constant of the circuit?

4) Consider 3 straight, infinitely long, parallel wires in the xz plane pointing along the $\hat{\mathbf{z}}$ direction with separation between adjacent wires of d. Each carries a current, I, has mass/length of m and has negligible radius.
a) Sketch the **B**-field between the wires
b) Calculate the two locations along the x-axis where the magnetic field is zero.
c) If the middle wire is rigidly displaced a very small distance ($x \ll d$) towards one of its neighbours while the other two wires are held fixed, qualitatively describe the resulting motion.
d) If, instead, the middle wire is rigidly displaced a very small distance ($y \ll d$) out of the plane of the other two wires, it follows simple harmonic motion of the form:

$$\ddot{y} = -ky$$

Find k for this case.

5) A long, solid, uniform cylindrical wire of radius a and relative permeability μ_r carries a steady free current, I.
a) Find the **B**-field and the **H**-field inside and outside of the wire.
b) By comparing this system with a *non-magnetic* wire of equivalent dimensions, determine the locations and directions of all bound currents in part (a) and qualitatively justify the similarities and differences between the results for the magnetic and non-magnetic case.