Formula Sheet 2 – Additional Formulae to Those Given In the Inside Cover of Griffiths

(When quoting a formula from here write down **the** formula **and** its number)

1. DEFINITIONS

Source charges & currents usually denoted by primes:

$$\vec{\wp}_{ij} = \vec{r}_j\,' - \vec{r}_i\,' \qquad (1.1)$$

$$\vec{\wp}_i = \vec{r} - \vec{r}_i\,' \qquad (1.2)$$

$$\vec{\wp} = \vec{r} - \vec{r}\,' \qquad (1.3)$$

2. ELECTROSTATICS

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\wp_{12}^{\,2}}\hat{\wp}_{12} \qquad (2.1)$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\wp_i^{\,2}}\hat{\wp}_i \qquad (2.2)$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}\,')}{\wp^2}\hat{\wp}\,d\tau' = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma(\vec{r}\,')}{\wp^2}\hat{\wp}\,da' = \frac{1}{4\pi\varepsilon_0}\int_P \frac{\lambda(\vec{r}\,')}{\wp^2}\hat{\wp}\,dl' \qquad (2.3)$$

$$\oint \vec{E}(\vec{r})\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} \qquad \vec{\nabla}\cdot\vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0} \qquad (2.4)$$

$$\oint \vec{E}(\vec{r}).d\vec{l} = 0 \qquad \vec{\nabla}\times\vec{E}(\vec{r}) = \vec{0} \qquad (2.5)$$

$$V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot d\vec{l} \qquad \vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r}) \qquad (2.6)$$

$$\nabla^2 V(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon_0} \qquad (2.7)$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\wp_i} \qquad (2.8)$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}\,')}{\wp}\,d\tau' = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma(\vec{r}\,')}{\wp}\,da' = \frac{1}{4\pi\varepsilon_0}\int_P \frac{\lambda(\vec{r}\,')}{\wp}\,dl' \qquad (2.9)$$

$$E^{\perp}_{away,medium1} - E^{\perp}_{towards,medium2} = \frac{\sigma}{\varepsilon_0} \qquad \vec{E}^{\parallel}_{medium\,1} = \vec{E}^{\parallel}_{medium2} \qquad (2.10)$$

$$W = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{j>i}^{n}\frac{q_i q_j}{\wp_{ij}} = \frac{1}{2}\sum_{i=1}^{n} q_i V(\vec{r}_i) \qquad (2.11)$$

$$W = \frac{1}{2}\int \rho(\vec{r})V(\vec{r})\,d\tau = \frac{\varepsilon_0}{2}\int_{all\,space}\left|\vec{E}(\vec{r})\right|^2 d\tau \qquad (2.12)$$

$$C = \frac{Q}{V} \qquad (2.13)$$

$$\mathrm{C} = \frac{\varepsilon_0 A}{d} \qquad (2.14)$$

$$W = \frac{1}{2}CV^2 \qquad (2.15)$$

3. SPECIAL TECHNIQUES

$$V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos(\theta)) \qquad (3.1)$$

$$P_0(x) = 1 \qquad P_1(x) = x$$
$$P_2(x) = (3x^2 - 1)/2 \qquad P_3(x) = (5x^3 - 3x)/2 \qquad (3.2)$$
$$P_4(x) = (35x^4 - 30x^2 + 3)/8 \qquad P_5(x) = (63x^5 - 70x^3 + 15x)/8$$

For arbitrary charge distribution at the origin (α between $\vec{r}\,'$ and $\vec{r}$):

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\alpha)\rho(\vec{r}\,')\,d\tau' = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r} + \frac{\vec{p}\cdot\hat{r}}{r^2} + \dots\right] \qquad (3.3)$$

$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i\,' \qquad (3.4)$$

$$\vec{p} = \int \vec{r}\,'\rho(\vec{r}\,')\,d\tau' \qquad (3.5)$$

$$\vec{E}_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{\wp^3}\left[3(\vec{p}.\hat{\wp})\hat{\wp} - \vec{p}\right] \qquad (3.6)$$

4. STATIC ELECTRIC FIELDS IN MATTER

$$\vec{p} = \alpha\,\vec{E} \qquad (4.1)$$

$$\vec{N} = \vec{p}\times\vec{E} \qquad (4.2)$$

$$\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E} \qquad (4.3)$$

$$U = -\vec{p}\cdot\vec{E} \qquad (4.4)$$

$$\sigma_b(\vec{r}_{surface}) = \vec{P}(\vec{r}_{surface})\cdot\hat{n} \qquad (4.5)$$

$$\rho_b(\vec{r}) = -\vec{\nabla}\cdot\vec{P}(\vec{r}) \qquad (4.6)$$

$$\vec{D}(\vec{r}) \equiv \varepsilon_0\vec{E}(\vec{r}) + \vec{P}(\vec{r}) \qquad (4.7)$$

$$\oint \vec{D}(\vec{r})\cdot d\vec{a} = Q_{free,enc} \qquad \vec{\nabla}\cdot\vec{D}(\vec{r}) = \rho_{free}(\vec{r}) \qquad (4.8)$$

$$D^{\perp}_{away,medium1} - D^{\perp}_{towards,medium2} = \sigma_{free} \qquad (4.9)$$

$$\vec{D}^{\parallel}_{medium\,1} - \vec{D}^{\parallel}_{medium2} = \vec{P}^{\parallel}_{medium\,1} - \vec{P}^{\parallel}_{medium2} \qquad (4.10)$$

$$\vec{P}(\vec{r}) = \varepsilon_0\chi_e(\vec{r})\vec{E}(\vec{r}) \qquad \text{(if linear dielectric, and also ↓)} \qquad (4.11)$$

$$\vec{D}(\vec{r}) = \varepsilon_0\varepsilon_r(\vec{r})\vec{E}(\vec{r}) = \varepsilon_0(1 + \chi_e(\vec{r}))\vec{E}(\vec{r}) \qquad (4.12)$$

$$W_{\text{tot stored}} = \frac{1}{2}\int_{all\,space}\vec{D}(\vec{r})\cdot\vec{E}(\vec{r})\,d\tau = \frac{\varepsilon_0}{2}\int_{all\,space}\varepsilon_r(\vec{r})\left|\vec{E}(\vec{r})\right|^2 d\tau \qquad (4.13)$$

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5. MAGNETOSTATICS

$$\vec{F} = q(\vec{v} \times \vec{B}) \tag{5.1}$$

$$\vec{F} = \int \vec{I} \times \vec{B}\, dl = \int (\vec{K} \times \vec{B})\, da = \int (\vec{J} \times \vec{B})\, d\tau \tag{5.2}$$

$$\vec{I} = \lambda \vec{v} \tag{5.3}$$

$$\vec{K} = \sigma \vec{v} \qquad I = \int K\, dl_\perp \tag{5.4}$$

$$\vec{J} = \rho \vec{v} \qquad I = \int \vec{J}.d\vec{a} \tag{5.5}$$

$$\vec{\nabla} \cdot \vec{J}(\vec{r},t) = -\frac{\partial \rho(\vec{r},t)}{\partial t} \tag{5.6}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} dl' \times \hat{\wp}}{\wp^2} = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}\,') \times \hat{\wp}}{\wp^2} da' = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times \hat{\wp}}{\wp^2} d\tau' \tag{5.7}$$

$$\oint \vec{B}(\vec{r}) \cdot d\vec{l} = \mu_0 I_{enc} \qquad \vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r}) \tag{5.8}$$

$$\oint \vec{B}(\vec{r}) \cdot d\vec{a} = 0 \qquad \vec{\nabla} \cdot \vec{B}(\vec{r}) = 0 \tag{5.9}$$

$$\vec{B}(\vec{r}) = \vec{\nabla} \times \vec{A}(\vec{r}) \tag{5.10}$$

$$\nabla^2 \vec{A}(\vec{r}) = -\mu_0 \vec{J}(\vec{r}) \qquad \text{(Coulomb gauge, and also ↓)} \tag{5.11}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{\wp} d\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}\,')}{\wp} da' = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}\,')}{\wp} dl' \tag{5.12}$$

$$B^\perp_{away,medium1} - B^\perp_{towards,medium2} = 0 \tag{5.13}$$

$$\vec{B}^\parallel_{medium\,1} - \vec{B}^\parallel_{medium2} = \mu_0 (\vec{K} \times \hat{n}_{2\to1}) \tag{5.14}$$

For an arbitrary current distribution at the origin and α the angle between $\vec{r}$ and $\vec{r}\,'$:

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\alpha)\, d\vec{l}\,' = \frac{\mu_0}{4\pi}\left[\frac{\vec{m} \times \hat{r}}{r^2} + ...\right] \tag{5.15}$$

$$\vec{m} = I\, \vec{a} \tag{5.16}$$

6. STATIC MAGNETIC FIELDS IN MATTER

$$\vec{N} = \vec{m} \times \vec{B} \tag{6.1}$$

$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B}) \tag{6.2}$$

$$U = -\vec{m} \cdot \vec{B} \tag{6.3}$$

$$\vec{J}_b(\vec{r}) = \vec{\nabla} \times \vec{M}(\vec{r}) \tag{6.4}$$

$$\vec{K}_b(\vec{r}_{surface}) = \vec{M}(\vec{r}_{surface}) \times \hat{n} \tag{6.5}$$

$$\vec{H}(\vec{r}) = \frac{\vec{B}(\vec{r})}{\mu_0} - \vec{M}(\vec{r}) \tag{6.6}$$

$$\oint \vec{H}(\vec{r}) \cdot d\vec{l} = I_{free,enc} \qquad \vec{\nabla} \times \vec{H}(\vec{r}) = \vec{J}_{free}(\vec{r}) \tag{6.7}$$

$$(\vec{H} + \vec{M})^\perp_{away,medium1} - (\vec{H} + \vec{M})^\perp_{towards,medium2} = 0 \tag{6.8}$$

$$\vec{H}^\parallel_{medium\,1} - \vec{H}^\parallel_{medium2} = \vec{K}_{free} \times \hat{n}_{2\to1} \tag{6.9}$$

$$\vec{M} = \chi_m \vec{H} \qquad \text{(if linear material, and also ↓)} \tag{6.10}$$

$$\vec{B}(\vec{r}) = \mu_0 \mu_r \vec{H}(\vec{r}) = \mu_0 (1 + \chi_m) \vec{H}(\vec{r}) \tag{6.11}$$

7. ELECTRODYNAMICS

$$\vec{J}(\vec{r}) = \sigma \vec{E}(\vec{r}) \tag{7.1}$$

$$\left.\begin{aligned} \varepsilon &= \oint \vec{f}_s \cdot d\vec{l} \\ \Phi &= \int \vec{B} \cdot d\vec{a} \end{aligned}\right\} \qquad \varepsilon = -\frac{d\Phi}{dt} \tag{7.2}$$

$$\oint \vec{E}(\vec{r},t) \cdot d\vec{l} = -\frac{d\Phi}{dt} \qquad \vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t} \tag{7.3}$$

$$\Phi_1 = LI_1 + \sum_{j \neq 1} M_{1j} I_j \qquad M_{1j} = \frac{\mu_0}{4\pi} \oint\oint \frac{d\vec{l}_1 \cdot d\vec{l}_j}{\wp_{1j}} \tag{7.4}$$

$$W = \frac{1}{2} L I^2 \tag{7.5}$$

$$W = \frac{1}{2\mu_0} \int_{all\,space} |\vec{B}(\vec{r})|^2 d\tau \tag{7.6}$$

$$\vec{\nabla} \cdot \vec{E}(\vec{r},t) = \frac{\rho(\vec{r},t)}{\varepsilon_0} \qquad \vec{\nabla} \times \vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}$$
$$\vec{\nabla} \cdot \vec{B}(\vec{r},t) = 0 \qquad \vec{\nabla} \times \vec{B}(\vec{r},t) = \mu_0 \vec{J}(\vec{r},t) + \mu_0 \varepsilon_0 \frac{\partial \vec{E}(\vec{r},t)}{\partial t} \tag{7.7}$$