

Midterm: Monday Feb 12, 2024

Name: _____

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes

PART 1 – answer questions **1-8** in pencil/pen here or in your exam booklet and **then** use your scratch card: one scratch = **100%**; two scratches = **50%**; three scratches = **25%** (part 1 total is 50%)

PART 2 – answer questions **9 & 10** in the exam booklet provided (each question here is worth 25%)

Qu's 1-2) Consider the *isothermal compressibility*, κ_T , of both an ideal gas and a van der Waals gas.

1) Which of the following expressions is the isothermal compressibility of an *ideal gas* in terms of its state variables?

A. nPV/R

B. $1/nP$

C. V/RT

D. $1/T$

E. $1/P$

2) Though derivation, or by employing dimensional analysis and recognizing which state variables scale with the number of moles (called *extensive* variables) and which don't (called *intensive* variables), which of the following expressions might plausibly be κ_T for a van der Waals gas?

A. $\frac{nR}{V-nb}$ B. $\frac{\left(1-\frac{nb}{V}\right)}{P+\frac{2n^2ab}{V^3}}$ C. $\frac{-nRT}{(V-nb)^2} + \frac{2n^2a}{V^3}$ D. $\frac{\left(1-\frac{nb}{V}\right)}{P-\frac{n^2a}{V^2} + \frac{2n^3ab}{V^3}}$ E. $\frac{\left(1-\frac{b}{V}\right)}{P-\frac{n^2a}{V^2} + \frac{2n^3ab}{V^3}}$

3) Liquid nitrogen is commonly used in hospitals, and it needs to be kept at -196°C . What is the *theoretical maximum* coefficient of performance for a refrigerator that can maintain this temperature if the refrigerator is placed in a room held at 20°C ? [Hint: pick a Carnot refrigerator for this].

A. 0.66

B. 0.56

C. 0.46

D. 0.36

E. 0.26

Qu's 4–6: We wish to compare the time taken to boil 1 kg of untreated tap water with the time taken to boil a solution made from 1 kg of tap water with salt *added to it* as per the table below. Assume a heating element with output of 1200 W, that no heat is lost, and there is negligible evaporation. Consider the following values, which we shall assume to remain constant during the heating:

	Untreated tap water	Tap water with salt added (concentration \approx that of seawater)
Mass of salt added	0 g salt per kg water	35 g salt per kg water
Density	998 kg/m ³	1025 kg/m ³
Specific heat, c_p	4186 J/kg/°C	3850 J/kg/°C
Boiling point at 1 atm	100 °C	102 °C

4) Based on the table above, what can be concluded about the effect of adding salt to tap water to produce a concentration of approximately that of seawater?

- A. The mass *increases* by 3.5% and the total volume *decreases* by $\approx 0.8\%$.
- B. The mass stays the same and the total volume *decreases* by $\approx 0.8\%$.
- C. The mass *increases* by 3.5% and the total volume *increases* by $\approx 0.8\%$.
- D. The mass stays the same and the total volume *decreases* by $\approx 2.7\%$.
- E. The mass *increases* by 8% and the total volume *decreases* by $\approx 0.8\%$.

5) How long does it take to bring to a boil 1 kg of *untreated* tap water, starting from a temperature of 22 °C?

- A. 272 s B. 256 s C. 280 s D. 250 s E. 278 s

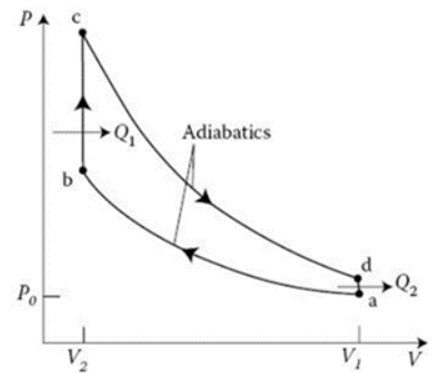
6) How does the time found in Question 5 compare with the time it takes to bring to a boil the solution of water and added salt described in the table, starting from the same temperature?

- A. The water/salt solution boils *more slowly*, by between 4 and 8 sec.
- B. The water/salt solution boils *more quickly*, by between 4 and 8 sec.
- C. The water/salt solution boils *more slowly*, by between 8 and 12 sec.
- D. The water/salt solution boils *more quickly*, by between 8 and 12 sec.
- E. The water/salt solution boils *more slowly*, by between 0 and 4 sec.

Qu’s 7–8: A simplified description of a gasoline engine, with air as the working substance, is shown. By treating air as an ideal diatomic gas, the theoretical efficiency of the engine can be found to be

$$\eta_{\text{gas}} = 1 - \left(\frac{V_2}{V_1} \right)^{\gamma-1}, \text{ with } V_1/V_2 \text{ called the } \textit{compression ratio}.$$

7) What is the efficiency of this engine for a compression ratio of 5?



- A. 0.47
- B. 0.54
- C. 0.68
- D. 0.86
- E. 0.92

8) Suppose you wish to write the theoretical efficiency of this gasoline engine in terms of the *temperature* of the air at various points in the cycle. Which of the following is the correct expression for its efficiency?

- A. $\eta_{\text{gas}} = 1 - \frac{T_d}{T_b}$
- B. $\eta_{\text{gas}} = 1 - \left(\frac{T_a}{T_b} \right)^{\gamma-1}$
- C. $\eta_{\text{gas}} = 1 - \frac{T_a}{T_c}$
- D. $\eta_{\text{gas}} = 1 - \left(\frac{T_a}{T_c} \right)^{\gamma-1}$
- E. $\eta_{\text{gas}} = 1 - \frac{T_a}{T_b}$

PART II –answer both questions in the exam booklet provided

9. a) A hypothetical substance has an isothermal compressibility $\kappa_T = an^2 / V^2$ and a volume expansion coefficient $\beta = 3nbT^2 / V$. By writing the general expression for dV , given that $V(T, P)$, find the equation of state for this substance in the form $f(V, T, P) = 0$.

b) Refrigerators are known to make use of the *Joule-Kelvin effect*.

(i) Explain with the help of a diagram what is meant by the Joule-Kelvin effect, and describe how this enables a region of space to be cooled (you need only focus on the Joule-Kelvin part of a refrigerator).

(ii) A van der Waals gas in the low pressure regime satisfies the following:

$$\mu_{JK} \approx \frac{1}{c_p} \left(\frac{2a}{RT} - b \right)$$

The van der Waals parameters for argon are $a = 0.1358 \text{ Pa m}^6 / \text{mol}^2$ and $b = 3.2 \times 10^{-5} \text{ m}^3 / \text{mol}$. By approximating argon to be an ideal gas for the purposes of finding c_p , find μ_{JK} for argon at 300 K in the low pressure regime, and determine how much its temperature would drop due to a pressure decrease of 0.5 atm.

10. a) An ideal gas with adiabatic exponent γ is compressed adiabatically from an initial state (P_i, V_i) to a final state (P_f, V_f) . Show that the work done on the gas in this process is

$$W = \frac{P_i V_i}{\gamma - 1} \left[\left(\frac{V_i}{V_f} \right)^{\gamma - 1} - 1 \right]$$

b) Find the work done on one mole of nitrogen (N_2), initially at 3 atm and 350 K, if it is compressed to half its volume, assuming it behaves as an ideal gas.