Trent University: PHYS 2700H – Thermal Physics 2022 – 2023

Midterm: Wednesday Feb 15, 2023 **Name:__________________**

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes

PART 1 – answer questions **1-8** in pencil/pen here or in your exam booklet **then** use your scratch card: One scratch = **100%**; two scratches **50%**; three scratches **33%**; four scratches **25%** (part 1 total is 50%)
PART 2 – answer questions **9** & **10** in the exam booklet provided (each question here is worth 25%)

Qu's 1 and 2: We apply a mathematical expression from Appendix B in Rex's *Thermal Physics*.

1) Suppose there exists an algebraic relationship between the three variables r, c, s. Which of the following then holds?

A. $\left(\frac{\partial r}{\partial c}\right)_s \left(\frac{\partial c}{\partial s}\right)_r = -\left(\frac{\partial r}{\partial s}\right)_c$ **B.** $\left(\frac{\partial r}{\partial c}\right)_s \left(\frac{\partial c}{\partial s}\right)_r = \left(\frac{\partial s}{\partial r}\right)_c$ **C.** $\left(\frac{\partial r}{\partial c}\right)_s \left(\frac{\partial c}{\partial s}\right)_r = -\left(\frac{\partial s}{\partial r}\right)_c$

D. $\left(\frac{\partial r}{\partial c}\right)_s \left(\frac{\partial c}{\partial s}\right)_r = \left(\frac{\partial r}{\partial s}\right)_c$ **E.** $\left(\frac{\partial r}{\partial c}\right)_s \left(\frac{\partial s}{\partial r}\right)_c = -\left(\frac{\partial c}{\partial s}\right)_r$

2) If we consider the equation of state for an ideal gas (namely, $PV = nRT$) and treat variables *r* as *P*, *c* as *V*, and *s* as *T*, which of the following expressions is on the left hand side (and therefore also the right hand side) of the correct answer to Question 1?

A. $-\frac{nR}{V}$ **B.** P **C.** T **D.** nRT **E.** $\frac{V}{nR}$

3) For an ideal gas at room temperature of $20\ ^\circ\text{C}$ and atmospheric pressure, which of the following numerical values are the correct: a) *isothermal compressibility*, and b) *volume thermal expansion coefficient*, assuming usual S.I. units, such as pressure in Pa, volume in m^3, temperature in K, etc.?

A. a) 1.013×10^{5} b) 0.04

B. a) 9.87×10^{-6} b) 0.04

C. a) 1.32×10^{-3} b) 3.41×10^{-3}

D. a) 9.87×10^{-6} b) 3.41×10^{-3}

E. a) 1.013×10^{5} b) 3.41×10^{-3}

Qu's 4 – 5: A common coolant within hospitals is liquid nitrogen, which needs to be kept at a temperature of $77\ \text{K}$.

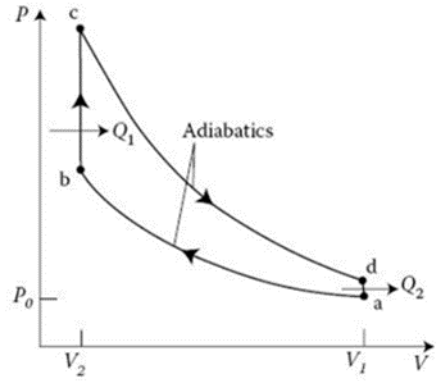
4) What is the *maximum* possible coefficient of performance for a refrigerator in a room at $20\ ^\circ\text{C}$ that can maintain this temperature? [Hint: pick a Carnot refrigerator for this].

A. 0.36 **B.** 0.46 **C.** 0.66 **D.** 0.56 **E.** 0.26

5) To obtain even colder temperatures for research laboratories liquid helium is required. From the maximum possible coefficient of performance for a refrigerator within a lab at $20\ ^\circ\text{C}$ that can maintain a cylinder of helium at a temperature of $4.2\ \text{K}$, what power is required by the refrigerator if 300 W of heat constantly leaks into the helium cylinder?

A. 2 kW **B.** 5 kW **C.** 13 kW **D.** 21 kW **E.** 42 kW

Qu's 6 – 8: A simplified description of a gasoline engine, with air as the working substance, is shown. By treating air as an ideal diatomic gas, the theoretical efficiency of the engine can be found to be $\eta_{\text{gas}} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$, with V_1 / V_2 called the *compression ratio*.



6) What is the efficiency of this engine for a compression ratio of 7?

A. 0.47 **B.** 0.54 **C.** 0.68 **D.** 0.86 **E.** 0.92

7) Suppose you wish to write the theoretical efficiency of this gasoline engine in terms of the *temperature* of the air at various points in the cycle. Which of the following is the correct expression for its efficiency?

A. $\eta_{\text{gas}} = 1 - \frac{T_{\text{d}}}{T_{\text{b}}}$ **B.** $\eta_{\text{gas}} = 1 - \left(\frac{T_{\text{a}}}{T_{\text{b}}}\right)^{\gamma-1}$ **C.** $\eta_{\text{gas}} = 1 - \frac{T_{\text{a}}}{T_c}$ **D.** $\eta_{\text{gas}} = 1 - \left(\frac{T_{\text{a}}}{T_{\text{c}}}\right)^{\gamma-1}$ **E.** $\eta_{\text{gas}} = 1 - \frac{T_{\text{a}}}{T_{\text{b}}}$

8) Suppose a Carnot engine operates between the *minimum* and *maximum* temperatures of this gasoline engine. What would be the efficiency of this Carnot engine?

A. $\eta_C = 1 - \frac{T_{\text{d}}}{T_{\text{b}}}$ **B.** $\eta_C = 1 - \left(\frac{T_{\text{a}}}{T_{\text{b}}}\right)^{\gamma-1}$ **C.** $\eta_C = 1 - \frac{T_{\text{a}}}{T_c}$ **D.** $\eta_C = 1 - \left(\frac{T_{\text{a}}}{T_{\text{c}}}\right)^{\gamma-1}$ **E.** $\eta_C = 1 - \frac{T_{\text{a}}}{T_{\text{b}}}$

PART II –answer <u>both</u> questions in exam booklet provided

9. a) State the two assumptions about the behaviour of the atoms or molecules in a gas that define an *ideal gas*, and write down its equation of state.
b) Show that the equation of state for a *van der Waals gas*, which is a good representation of a real gas, can take the following form:

$$P = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2} \quad .$$

c) From the equation above, find the conditions on V and T under which a van der Waals gas increasingly behaves like an ideal gas. Sketch on a PV diagram an arbitrary isotherm for an ideal gas and, from the equation above, determine and sketch also the corresponding isotherm for a van der Waal gas. Identify two differences in the behaviour of these isotherms.
d) For a real gas there is a particular value of both V and T, called the *critical volume,* V_c, and *critical temperature,* T_c, for which the T_c isotherm has a point of inflection on the PV diagram (i.e. zero first derivative and zero second derivative). Show that for a van der Waals gas these two quantities are given by:

$$V_c = 3nb \qquad \text{and} \qquad T_c = \frac{8a}{27Rb}$$

e) The van der Waals parameters for argon are: $a = 0.1358\ \text{Pa m}^6/\text{mol}^2$ and $b = 3.2\times10^{-5}\ \text{m}^3/\text{mol}$. Find for one mole of argon the corresponding values of V_c and T_c.

10. a) An ideal gas with adiabatic exponent γ is compressed adiabatically from an initial state (P_i, V_i) to a final state (P_f, V_f). Show that the work done on the gas in this process is

$$W = \frac{P_i V_i}{\gamma - 1}\left[\left(\frac{V_i}{V_f}\right)^{\gamma-1} - 1\right]$$

b) Find the work done on the gas if one mole of nitrogen gas (N_2), initially at $2\ \text{atm}$ and $300\ \text{K}$, is compressed to half its volume.