Midterm: Monday February 12, 2018

Answer all four questions.
Each question carries equal marks. Show all working.
Allowed: 1 hour 50 mins. Calculator, formula sheet (given)

1. You wish to cool a cup of tea of mass 200 g, initially at $100\,^\circ\mathrm{C}$.
a) You add 100 g of *supercooled water*, initially at $-5\,^\circ\mathrm{C}$. Assuming the only heat transfer is between the tea and the water and that $c_{\text{tea}} \approx c_{\text{water}}$, what is the final temperature at thermal equilibrium?
b) What is the total heat flow from the tea to the water?
c) If instead you cool the tea with *ice* at $-5\,^\circ\mathrm{C}$ instead of water, how much ice is required to reach the same equilibrium temperature as in part (a)?

2. a) Show that the isothermal compressibility of an *ideal gas* is $1/P$, where P is the pressure. Briefly explain whether this is consistent with what you physically expect and why.
b) Determine the isothermal compressibility for a *van der Waals gas*, and show it reduces to the expression for an ideal gas when the parameters take the appropriate values.

3. a) Show that for an ideal gas the slope of an adiabat at a particular P and V on a PV diagram is γ times the slope of an isotherm passing through that point.
b) Write down the *cyclic relation* for partial differentials using the continuous variables. P, V, T. Demonstrate that this relationship holds for these variables when related by the ideal gas equation.

4. 2.5 moles of a monatomic ideal gas is initially at $300\,\mathrm{K}$ and $P = 1.0\,\mathrm{atm}$. The gas is then taken on a three step cycle: (i) the pressure and volume are increased with P proportional to V, until 2.0 atm; then (ii) pressure is reduced at constant volume to 1.0 atm; (iii) the volume is reduced at constant pressure until the initial state is reached.
a) Find the internal energy and volume occupied by the gas in its initial state.
b) Find $\Delta U, W$ and Q for each step, where W is the work done ***on***, and Q is the heat flow ***in*** to the gas.
c) Find the net values of $\Delta U, W$ and Q for the entire cycle. Do you observe a difference between the magnitudes of W and Q for the cycle? Explain why this is or is not the case.
d) Find the efficiency of a heat engine that operates using this cycle
e) What is the maximum efficiency of a heat engine that operates between the same minimum and maximum temperatures?

Useful constants:

Latent heat of fusion of water = $334\,\mathrm{kJ/kg}$
Specific heat of ice = $2.08\,\mathrm{kJ/kg/K}$
Specific heat of water = $4.18\,\mathrm{kJ/kg/K}$