

**Final Exam: Saturday Apr 18, 2026** Name: \_\_\_\_\_

**Allowed: Formula sheet (given), calculator, 2 ½ hours**

PART 1 – answer questions 1-7 in the exam booklet provided and **then** use your scratch card:

One scratch = **100%**; two scratches **33%**; three scratches **25%** (part 1 total is 40%)

PART 2 – answer questions 8–10 in the exam booklet provided (each question here is worth 20%)

**Qu’s 1-2:** Consider  $n$  moles of an ideal gas with adiabatic exponent  $\gamma$ , initially at  $P_i, V_i$  and  $T_i$ , which reversibly undergoes an adiabatic compression to  $P_f, V_f$  and  $T_f$ .

1) What is the final pressure of the gas,  $P_f$ , after this process in terms of the volume change?

- A.  $(V_f/V_i)^{\gamma-1} P_i$     B.  $(V_i/V_f)^{\gamma-1} P_i$     C.  $(V_i/V_f)^\gamma P_i$     D.  $(V_f/V_i)^\gamma P_i$     E.  $(V_i/V_f) P_i$

2) Which of the following equations correctly relates the final temperature and volume of the gas to its initial temperature and volume?

- A.  $T_f^\gamma V_f = T_i^\gamma V_i$     B.  $T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$     C.  $T_f^\gamma V_f = T_i^\gamma V_i$     D.  $T_f^\gamma V_f = T_i^\gamma V_i$     E.  $T_f V_f^{\frac{\gamma-1}{\gamma}} = T_i V_i^{\frac{\gamma-1}{\gamma}}$

**Qu’s 3-4)** Consider 3 moles of helium with initial volume and pressure of  $V_1 = 64 \text{ dm}^3$  and  $P_1 = 1 \text{ atm}$ , respectively. We shall treat the helium as an ideal monatomic gas. The gas undergoes an *isobaric expansion* to twice its original volume.

3) Which of the following best represents the change in internal energy of the gas?

- A. 6.0 kJ    B. 7.3 kJ    C. 8.8 kJ    D. 9.7 kJ    E. 4.7 kJ

4) Which of the following best represents the change in enthalpy of the gas?

- A. 15.2 kJ    B. 28.5 kJ    C. 23.6 kJ    D. 19.4 kJ    E. 16.2 kJ

5) Suppose a fridge runs on 60 W of electrical power, while releasing 360 W of heat into the basement. Which of the following is the *coefficient of performance* of this fridge?

- A. 14                      B. 5                      C. 0.86                      D. 1.2                      E. 0.17

**Qu's 6–7:** A fairly elementary calculation of the total number of ways to assign the  $N$  indistinguishable particles of an ideal monatomic gas with volume  $V$  and internal energy  $U = N \times \frac{1}{2} m \overline{v^2}$  among the permitted cells of phase space gives  $\left( \frac{eVp_{rms}^3}{Nh^3} \right)^N$ , where  $e$  is Euler's number (2.718...) and  $rms$  denotes 'root mean square'.

6) Which of the following expressions is therefore the entropy of this gas, in terms of volume and temperature?

- A.  $S = Nk_B \ln \left[ \frac{eV}{N} \left( \frac{3mk_B T}{h^2} \right)^{5/2} \right]$       B.  $S = Nk_B \ln \left[ \frac{eV}{N} \left( \frac{3mk_B T}{h^2} \right)^{5/2} \right]$       C.  $S = nk_B \ln \left[ \frac{eV}{N} \left( \frac{3mk_B T}{h^2} \right)^{3/2} \right]$   
D.  $S = Nk_B \ln \left[ \frac{eV}{N} \left( \frac{3mk_B T}{h^2} \right)^{3/2} \right]$       E.  $S = nR \ln \left[ \frac{eV}{N} \left( \frac{3mk_B T}{h^3} \right)^{5/2} \right]$

7) A more-advanced calculation results in the *Sacker-Tetrode expression* for the entropy of an ideal monatomic gas:

$$S = Nk_B \ln \left[ \frac{V}{N} \left( \frac{2\pi mk_B T}{h^2} \right)^{3/2} \right] + \frac{5}{2} Nk_B$$

Using this expression, what is the entropy of one mole of  $^4\text{He}$  at 293 K and 1 atm?

- A. 92 J/K                      B. 104 J/K                      C. 115 J/K                      D. 126 J/K                      E. 146 J/K

**PART II – answer all three questions in exam booklet provided**

**8.** The equation of state for a stretched elastic band connects the tension  $F$ , the length  $L$ , and the temperature  $T$  according to:

$$F = aT \left[ \frac{L}{L_0} - \left( \frac{L_0}{L} \right)^2 \right],$$

where  $L_0$  is the unstretched length,  $T$  is in Kelvin, and  $a$  is a constant.

**(a)** Write down the cyclic relation between  $F$ ,  $L$ , and  $T$ , and from this obtain an expression for  $(\partial F / \partial T)_L$  in terms of two other partial derivatives.

**(b)** Make a large sketch of  $F(L)$  showing  $L$  ranging from  $L_0$  to  $4L_0$  for **two** temperatures:  $T_1$  and  $\frac{9}{4}T_1$ , and find for these two temperatures the values of  $F$  at  $2L_0$  and  $4L_0$ .

**(c)** An elastic band follows a cycle, going from point **a** at  $4L_0$  and  $T_1$ ; directly to point **b** at  $4L_0$  and  $\frac{9}{4}T_1$ ; then isothermally to point **c** at  $2L_0$ ; then directly back to **a**, etc. (Note that points **c** and **a** correspond to the same force). Label this cycle on your graph, and write down the integral in terms of  $a$ ,  $L_0$  and  $T_1$  (but you do not need to compute it) for the work done **by** the elastic band in going from **b** to **c**. (Note the infinitesimal work done *on* a stretched elastic band when it lengthens by  $dL$  is  $\delta W = FdL$ ).

**9. (a)** Explain the physical significance of the  $a$  and the  $b$  variables in the equation of state for a *van der Waals gas* and why these values are zero for an ideal gas.

**(b)** Sketch a  $PV$  diagram for a single substance that shows the regions corresponding to a vapour and a vapour-liquid mix. Label also the location of the *critical point*,  $C$ . Draw on this diagram isotherms for both an ideal gas and a van der Waal gas that pass slightly above the critical point, and explain the qualitative differences between the isotherms.

**(c)** The location of the critical point  $C$  can be found from the requirement that both the first and second derivative of  $P(V)$  for the critical isotherm is zero: namely  $(\partial P / \partial V)_{T_c} = 0$  and  $(\partial^2 P / \partial V^2)_{T_c} = 0$ .

Given this, find in terms of  $a$  and  $b$  the critical pressure and critical volume of a van der Waal gas.

**10.** Consider the Gibbs function,  $G$ , for a *closed* system.

**(a)** Determine from the appropriate thermodynamic identity an expression for the entropy and the volume of the system in terms of partial derivatives of  $G$ .

**(b)** Derive the corresponding Maxwell relation.

**(c)** How does the thermodynamic identity change if the system were instead *open*? Find an expression for the chemical potential of the system in terms of a partial derivative of  $G$ .

**(d)** If two open systems composed of the same single substance are free to exchange particles with each other, and system 1 has a chemical potential of  $\mu_1 = -0.646$  eV, while system 2 has a chemical potential of  $\mu_2 = -0.346$  eV, then which way, if at all, is there a net flow of particles?