

**Final Exam: Monday Apr 15, 2024** Name: \_\_\_\_\_

**Allowed: Formula sheet (given), calculator, 2½ hours**

PART 1 – answer questions **1-7** in pencil/pen in your exam booklet **then** use your scratch card:

One scratch = **100%**; two scratches **50%**; three scratches **25%** (part 1 total is 40%)

PART 2 – answer questions **8–10** in the exam booklet provided (each question here is worth 20%)

**Qu’s 1-5)** A gas of  $n$  moles of helium, comprising  $N$  atoms, initially at  $P_i$ ,  $V_i$  and  $T_i$ , is reversibly compressed until its volume is one-third of its original volume. Treat this gas as an ideal gas.

1) If the compression takes place isothermally, which of the following are the correct values of the final pressure and temperature?

- A.  $P_f = 3P_i$   
 $T_f = T_i$       B.  $P_f = 6.24P_i$   
 $T_f = 2.08T_i$       C.  $P_f = P_i$   
 $T_f = T_i/3$       D.  $P_f = 2.74P_i$   
 $T_f = 1.10T_i$       E.  $P_f = P_i$   
 $T_f = 3T_i$

2) If instead the compression takes place isobarically, which of the following are the correct values of the final pressure and temperature?

- A.  $P_f = 3P_i$   
 $T_f = T_i$       B.  $P_f = 6.24P_i$   
 $T_f = 2.08T_i$       C.  $P_f = P_i$   
 $T_f = T_i/3$       D.  $P_f = 2.74P_i$   
 $T_f = 1.10T_i$       E.  $P_f = P_i$   
 $T_f = 3T_i$

3) For the isobaric process, which of the following is the work done on the gas?

- A.  $-nRT_i \ln 3$       B.  $nRT_i \ln 3$       C.  $\frac{2}{3}P_iV_i$       D.  $1.62nRT_i$       E.  $-1.62nRT_i$

4) Using the expression we derived in class for the entropy of an ideal gas ( $S = nc_v \ln T + nR \ln V + ns_0$ ), or otherwise, which of the following is the entropy change of the gas for the isobaric process?

- A.  $\Delta S = -nR \ln 3$       B.  $\Delta S = nR \ln 3$       C.  $\Delta S = 1.62nR$       D.  $\Delta S = -\frac{3}{2}nR \ln 3$       E.  $\Delta S = -\frac{5}{2}nR \ln 3$

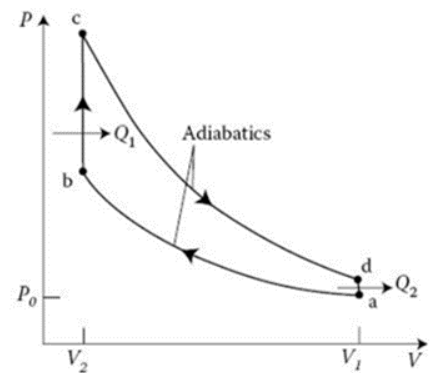
5) Which of the following expressions indicate how many more or fewer microstates  $\Omega$  are available to the gas at the end of the *isobaric* process than there were initially?

- A.  $\Omega_f = e^{-N \ln 3} \times \Omega_i$       B.  $\Omega_f = e^{-(5/2)N \ln 3} \Omega_i$       C.  $\Omega_f = e^{N \ln 3} \times \Omega_i$   
 D.  $\Omega_f = e^{1.62N} \times \Omega_i$       E.  $\Omega_f = nR \times \Omega_i$

Qu’s 6–7: A simplified description of a gasoline engine, with air as the working substance, is shown. By treating air as an ideal diatomic gas, the theoretical efficiency of the engine can be found to be

$$\eta_{\text{gas}} = 1 - \left( \frac{V_2}{V_1} \right)^{\gamma-1}, \text{ with } V_1/V_2 \text{ called the } \textit{compression ratio}.$$

6) What is the efficiency of this engine for a compression ratio of 5?



- A. 0.47      B. 0.54      C. 0.68      D. 0.86      E. 0.92

7) Suppose you wish to write the theoretical efficiency of this gasoline engine in terms of the *temperature* of the air at various points in the cycle. Which of the following is the correct expression for the efficiency?

- A.  $\eta_{\text{gas}} = 1 - \frac{T_d}{T_b}$       B.  $\eta_{\text{gas}} = 1 - \frac{T_a}{T_b}$       C.  $\eta_{\text{gas}} = 1 - \frac{T_a}{T_c}$       D.  $\eta_{\text{gas}} = 1 - \left( \frac{T_a}{T_c} \right)^{\gamma-1}$       E.  $\eta_{\text{gas}} = 1 - \left( \frac{T_a}{T_b} \right)^{\gamma-1}$

**PART II – answer all three questions in exam booklet provided**

- 8) a) Show that the isothermal compressibility of an *ideal gas* is  $1/P$ , where  $P$  is the pressure. Briefly explain why this is compatible with what we understand by “an ideal gas”.
- b) Determine the isothermal compressibility for a *van der Waals gas*, and show it reduces to the expression for an ideal gas when the relevant parameters take the appropriate values.
- c) The van der Waals parameters for argon are  $a = 0.1358 \text{ Pa}\cdot\text{m}^6/\text{mol}^2$ ;  $b = 3.2 \times 10^{-5} \text{ m}^3/\text{mol}$ . By what approximate percentage does the isothermal compressibility of argon differ from that of an ideal gas at STP?

9) Consider distributing ten indistinguishable quanta of energy among **two** objects, each of which have eight distinct locations for the quanta.

- a) How many different ways are there to distribute the quanta so that *two* quanta are on one object and *eight* are on the other?
- b) Assuming all possible microstates are equally likely, how much more likely are you to find this system with *five* quanta of energy on each object?

10) We shall use here the following information for  $\text{H}_2\text{O}$  (note the units):

Latent heat of fusion at 1 atm pressure: 334 kJ/kg

Latent heat of vaporization at 1 atm pressure: 2257 kJ/kg

Specific volume of ice at 1 atm pressure and  $0^\circ\text{C}$  :  $1.0907 \text{ cm}^3/\text{g}$

Specific volume of water at 1 atm pressure and  $0^\circ\text{C}$  :  $1.0001 \text{ cm}^3/\text{g}$

Specific volume of water at 1 atm pressure and  $100^\circ\text{C}$  :  $1.043 \text{ cm}^3/\text{g}$

Specific volume of steam at 1 atm pressure and  $100^\circ\text{C}$  :  $1.6959 \text{ m}^3/\text{kg}$

- a) Sketch the  $P$ - $T$  diagram for  $\text{H}_2\text{O}$ , showing the three regions that represent ice, water, and steam, and draw and label the phase curves that separate these phases.
- b) Find the slopes of the two phase curves (in units of Pa/K) for fusion and for vaporization at 1 atm. Which curve has the greatest slope, and how much greater is this?
- c) Estimate the pressure change required for ice initially at  $\approx 1 \text{ atm}$  to change its melting point by  $10^\circ\text{C}$
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