

Final Exam: Monday April 16, 2018

Answer all seven questions.

Each question carries equal marks. Show all working.

Allowed: 3 hours. Calculator, graph paper (given), formula sheet (given)

1. a) The equation of state for a single volumetric substance can be generically written as $f(P, V, T) = 0$. Briefly explain what is meant by this equation and write down the relevant equation for an ideal gas.
- b) The equation of state for an elastic material can be similarly written as $f(F, L, T) = 0$. For an elastic band this can be written as:

$$F = aT \left[\frac{L}{L_0} - \left(\frac{L_0}{L} \right)^2 \right],$$

where L_0 is the original length, T is measured in Kelvin, and $a = 1.3 \times 10^{-2}$ N/K.

- i) Sketch in your exam booklet the graph of $F(L)$ for a fixed temperature T for a sensible range of L .
- ii) How much work is done on the elastic band when it is isothermally and reversibly stretched from an original length of 10 cm to a length of 20 cm at a temperature of 20 °C?

2. One mole of helium at P_i, V_i and T_i is reversibly compressed until its volume is one-third of its original volume. Treat this as an ideal gas. Answer the following questions in terms of P_i, V_i or T_i .

- a) If the compression takes place isothermally, what is the resulting pressure? Find ΔU , W , Q and ΔS for this process.
- b) If the compression takes place isobarically, what is the resulting temperature? Find ΔU , W , Q and ΔS for this process.
- c) If the compression takes place adiabatically, what is the resulting temperature and pressure? Find ΔU , W , Q and ΔS for this process.
- d) Sketch in your exam booklet each of these processes on a **single** PV diagram, and label each process.

[For this question above, equation 8.28 from Finn may help: $S = n(c_p \ln T - R \ln P + s_0)$]

3. Suppose we have two identical samples of a substance, a colder one at a temperature of T_1 K and a hotter one at T_2 K. The two samples are placed in thermal contact but otherwise isolated from the environment. We assume a constant heat capacity for each sample.

- a) What is the final temperature of the two samples?
- b) What is the change in entropy for each sample?
- c) What is the net change in entropy?
- d) What would happen to your answer in (c) if $T_1 < 0$? Is this physical?

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4. a) Show that the isothermal compressibility of an *ideal gas* is $1/P$, where P is the pressure. Briefly explain whether this is consistent with what you physically expect and why.
 b) Determine the isothermal compressibility for a *van der Waals gas*, and show it reduces to the expression for an ideal gas when the relevant parameters take the appropriate values.
 c) The van der Waals parameters for argon are $a = 0.1358 \text{ Pa}\cdot\text{m}^6/\text{mol}^2$; $b = 3.2 \times 10^{-5} \text{ m}^3/\text{mol}$. By what percentage does the isothermal compressibility of argon differ from that of an ideal gas at STP?

5. a) Deduce, showing your working, what equation 8.1 on the formula sheet simplifies to for n moles of an ideal gas.
 b) Derive **either** equation 8.1 **or** equation 8.3 from the formula sheet.

6. The table below contains some observations of the phase of a single substance at various temperatures and pressures. Each phase change experienced by this particular substance is *first-order*.

Temperature ($^{\circ}\text{C}$)	Pressure (atm)	Observed Phase
-78	0.5	Vapour
-78	1.1	Solid
-78	10	Solid
-57	4	Vapour
-57	6	Solid
-57	19	Solid
-50	6	Vapour
-50	7	Liquid
-50	15	Solid
-30	7	Vapour
-30	8	Liquid
-30	90	Liquid
-30	100	Solid

- a) Explain what is meant by a *first-order phase change*.
 b) Use the graph paper provided to plot and thus to understand this data. **Then** draw in your exam booklet an approximate, but large and well-labelled, phase diagram for this substance. Denote normal atmospheric pressure on your diagram.
 c) Which aspect of your phase diagram appears to be consistent with what you would expect from the Third Law of Thermodynamics, and why?

7. Consider distributing 16 physically-identical but distinguishable (i.e. can be labelled) particles among **two** distinct locations.

- a) How many different macrostates and how many total microstates are there for this system?
 b) Assuming all microstates are equally likely, how much more likely are you to find the system in the most probable macrostate compared to the least probable macrostate? Describe the properties of the most probable macrostate?
 c) What would be the change in the system's entropy if it moved from the least probable macrostate to the most probable macrostate?