Trent: PHYS 2620H – Atomic, Molecular and Nuclear Physics: 2015-2016

Midterm: Wednesday Feb 3, 2016 **Name**______________

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes

PART 1 – answer questions 1-8 in the spaces provided, **then** use your scratch card: one scratch **4** points; two scratches **1** point; three scratches **0.5** point (total for part 1 is worth 50%)

PART 2 – answer questions **9** & **10** in the exam booklet provided (each question is worth 25%)

Qu's 1 -2 The wave function for a particle of mass *m* in the ground state of a simple harmonic oscillator is: $\psi_0(x) = A\exp\left(-\left(km/\hbar^2\right)^{\frac{1}{2}} x^2/2\right)$

1) From which of the following integral equations can one obtain the *normalisation constant*, *A*?

A. $A^2 \int_{-\infty}^{\infty} e^{-\left(km/\hbar^2\right)x^2} dx = 1$ **B.** $A^2 \int_{-\infty}^{\infty} e^{-\left(km/\hbar^2\right)^{1/2} x^2} dx = 1$ **C.** $A \int_{-\infty}^{\infty} e^{-\left(km/\hbar^2\right)^{1/2} x^2} dx = 1$

D. $A^2 \int_{-\infty}^{\infty} e^{-\left(km/\hbar^2\right)x^4/4} dx = 1$ **E.** $\int_{-A}^{A} e^{-\left(km/\hbar^2\right)^{1/2} x^2} dx = 1$

2) What are the S.I. units of $\psi_0(x)$ given above?

A. $kgms^{-1}$ **B.** dimensionless **C.** kg^{-1} **D.** $m^{-1/2}$ **E.** m^{-1}

3) The wave function for a particle in the ground state of an infinite well of length *L* is given by

$$\psi(x) = \sqrt{2/L}\sin(\pi\, x/L)$$

What is the approximate probability of the particle lying within $\pm L/20$ of $x = L/4$ in this state?

A. 0.5 **B.** 0.4 **C.** 0.3 **D.** 0.2 **E.** 0.1

4) $Y_{m_\ell}^{\ell}(\theta,\phi) \equiv \Theta_{\ell,m_\ell}(\theta)\Phi_{m_\ell}(\phi)$ is an eigenfunction of both the $\hat{L}^2$ operator and the $\hat{L}_z$ operator. What are the respective eigenvalues of $\hat{L}^2$ and $\hat{L}_z$ for $(\ell, m_\ell) = (2,+2)$?

A. $2\hbar$ and $+2\hbar$, respectively **B.** $-2\hbar^2$ and $+2\hbar$, respectively **C.** $-6\hbar^2$ and $2\hbar$, respectively

D. $\sqrt{6}\,\hbar$ and $+2\hbar$, respectively **E.** $6\hbar^2$ and $+2\hbar$, respectively

5) In general, for which of the $\hat{L}^2$, $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ operators can a particle simultaneously have well-defined eigenvalues?

A. $\hat{L}^2$ and any **three** of $\hat{L}_x$, $\hat{L}_y$ or $\hat{L}_z$ **B.** $\hat{L}^2$ and any **two** of $\hat{L}_x$, $\hat{L}_y$ or $\hat{L}_z$
C. $\hat{L}^2$ and any **one** of $\hat{L}_x$, $\hat{L}_y$ or $\hat{L}_z$ **D.** **any** two of these four **E.** any **three** of these four

6) Assume the *orbital angular momentum quantum number* of an electron takes the value 2. Within a vector model picture what is the smallest possible angle between the vector $\vec{L}$ and the *z*-axis?

A. 18° **B.** 23° **C.** 27° **D.** 30° **E.** 35°

7) A CD has a moment of inertia $I = 2.5\times10^{-5}$ kg⋅m^2 and rotates at 400 rev/min. Using the relation from PHYS 1001H that $L = I\omega$ then what approximately is its orbital angular momentum quantum number?

A. 1.2×10^{32} **B.** 1.2×10^{31} **C.** 9.9×10^{30} **D.** 2.0×10^{30} **E.** 3.5×10^{15}

8) For the hydrogen atom in the ground state what is the probability of finding the electron anywhere in the range $\Delta r = 0.04\,a_0$ centred at $r = 1.5a_0$?

A. 0.046 **B.** 0.034 **C.** 0.022 **D.** 0.018 **E.** 0.012

PART II – answer both questions in exam booklet provided

9. a) Name and give the usual symbols for the quantum numbers required to specify an electronic state of a hydrogen atom (neglecting any effects of spin) and give the general relationships between the possible values of these quantum numbers. What physical property do each these quantum numbers reflect?
b) On the same graph and with the same axes carefully sketch the radial probability density, $P(r)$, for a 2p electron and a 3d electron. Briefly explain how you obtained each part of each sketch.
c) Calculate the average distance between the electron and the proton in the $2p_0$ state. Is this greater or less than the *most likely* distance where the electron would be found within a small radius Δr ?
d) The hydrogen atom emission spectrum results from transitions from higher to lower energy states. The transitions to each level of a particular n form a series of absorption lines, named *Lyman* (all transitions to $n = 1$), *Balmer* (all transitions to $n = 2$), *Paschen* (all transitions to $n = 3$), *Brackett*, etc. The Lyman series of transitions does not overlap the Balmer series in terms of frequency of light emitted. Which of these series is the first to overlap its neighbour? Demonstrate how you got your answer.

10. a) Write down the mathematical form of the following two equations in the most convenient coordinate scheme, including as much detail in your equation and/or supporting text as possible:
i) the time-independent Schrödinger equation for **one** particle of mass m in **one**-dimensional space experiencing a potential energy $U(x)$.
ii) the time-independent Schrödinger equation for **two** particles each of mass m in **three**-dimensional space experiencing a central potential energy $U(r_1, r_2)$.

b) It is a general property of eigenfunctions of the total energy operator that any two eigenfunctions corresponding to different eigenvalues are *orthogonal*. For example, in one-dimension, two solutions with different energies of the Schrödinger equation, labelled m and n, satisfy the following:

$$\int_{-\infty}^{\infty} \psi^*{}_m(x)\psi_n(x)dx = 0$$

By extending this concept of orthogonality to three-dimensions, verify that it is also satisfied by the two solutions of the hydrogen atom that are conventionally labelled $3p_{+1}$ and $2p_{+1}$.