Trent: PHYS 2620H – Atomic, Molecular and Nuclear Physics: 2014-2015

Midterm: Monday Feb 9, 2015

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes

Answer all of question one (worth 40% of the marks), and two out of the remaining three questions

Do all of this question:

1a) Give, with reasoning, the S.I. units of: i) a 1-dimensional, one-particle wave function, $\psi(x)$, and
ii) a 3-dimensional, one-particle wave function, $\psi(\vec{r})$.

b) Explain what is meant by the term *expectation value*. For a particle described by a generic 1-dimensional wave function $\psi(x)$, write down and simplify as much as possible the integral required to calculate the expectation value of the particle's linear momentum squared: $\langle p^2 \rangle$.

c) The spherical harmonic $Y^{\ell}_{m_\ell}(\theta,\phi) \equiv \Theta_{\ell,m_\ell}(\theta)\Phi_{m_\ell}(\phi)$ is an eigenfunction of both the $\hat{L}^2$ operator and the $\hat{L}_z$ operator. By recalling the general form of the eigenvalue equations for each operator determine their eigenvalues when $\ell = 4$ and $m_\ell = 3$.

d) For the hydrogen atom in the ground state the radial probability density is given by

$$P(r) = \frac{4}{a_0^3} r^2 \exp(-2r/a_0).$$

Estimate the probability of finding the electron in the range $\Delta r = 0.04\,a_0$ centred at $r = 3a_0$.

Do two out of these three remaining questions:

2. To a good approximation the hydrogen chloride molecule (HCl) behaves vibrationally as a quantum harmonic oscillator of spring constant 480 N/m and with a mass given by that of the hydrogen atom. Assume that it is in its ground state:
a) Sketch on the same plot the potential energy function, and the ground state wave function of this system. Find the location of the classical turning points in the ground state and denote them on the plot.
b) What wavelength of light would excite this molecule to its next-highest vibrational energy state, and in which region of the electromagnetic spectrum does it lie?

3. Consider a hydrogen atom in a $2p_y$ orbital, with wave function defined by:

$$\psi_{2p_y} = \frac{1}{\sqrt{2}}\left(\psi_{2p,+1} - \psi_{2p,-1}\right)$$

a) Show explicitly that $\psi_{2p,+1}$ is an eigenfunction of both $\hat{L}^2$ and $\hat{L}_z$ and determine the eigenvalues.

b) Show whether or not ψ_{2p_y} is an eigenfunction of $\hat{L}_z$. What is the significance of this result?

4. The course textbook includes the following formula within the chapter on the hydrogen atom:

$$R_{n,n-1}(r) \propto r^{n-1} \exp(-r/na_0)$$

a) Explain in a few sentences what is meant by this equation and each term in it, and use your formula sheet to verify it for $n = 1, 2$ and 3.
b) For $n = 2$, find the corresponding *radial probability density*, $P(r)$, and determine the position of its maximum. What is significant about this position?