Trent: PHYS 2620H – Atomic, Molecular and Nuclear Physics: 2010-2011

Midterm: Tuesday Feb 15, 2011

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes
Answer all of Part 1 (worth 40% of the marks), and two out of the remaining three questions

Advice: Don't cram your answers into too small a space – explain your answers

Do all of this first question:

1 a) From the formula sheet find the radial probability density, $P(r)$, for a hydrogen atom in the $n = 2, \ell = 1, m_\ell = 0$ state (a $2p_z$ orbital). Make a rough sketch of this function, find the position of its maximum, and estimate the probability of finding the electron in the range $\Delta r = 0.04a_0$ at $r = 2a_0$.

b) Using the $\{n, \ell, m_\ell, m_s\}$ set of quantum numbers, write down all possible distinct quantum states for the hydrogen atom when $n = 4$. How many states are there in total?

c) Carefully, but using only a few sentences with a diagram or two, describe the principle behind the Stern-Gerlach experiment. Suppose it were possible to send a beam of hydrogen atoms in the 3d state into such an experiment. If there were no electron spin, how many peaks of atom intensity are expected to be observed?

d) The functions $\Theta_{\ell,m_\ell}(\theta)\Phi_{m_\ell}(\phi)$ given on your formula sheet are eigenfunctions of both the $\hat{L}^2$ operator and the $\hat{L}_z$ operator. What are the respective eigenvalues for $\ell = 3$ and $m_\ell = 2$?

Do two out of these three remaining questions:

2. a) With energy on the vertical axis and ℓ on the horizontal axis, sketch and label all possible hydrogen atom energy levels from $n = 1$ up to $n = 3$, neglecting the effects of electron spin. Label the energies of all states.
b) Over what range of n is the spacing between the hydrogen atom energy levels with principal quantum numbers n and $n + 1$ less than 0.01 eV?

3. Write down the functional forms of the wave function $\psi(\vec{r})$ for an electron in the 3d, $m_\ell = +1$ state of the hydrogen atom, neglecting the effect of electron spin. Show that the expectation value of r, the distance between the electron and the nucleus, is $10.5\,a_0$.

4. Calculate the probability of finding the electron in the hydrogen atom somewhere inside a cone of $\theta = 23.5°$ ("arctic polar region") if the electron is in the state $n = 2$, $\ell = 1$, $m_\ell = 0$ (i.e. in a a $2p_z$ orbital).