Physics 203H Midterm, February 27th, 2006

Allowed: 1 single-sided sheet of 8½×11" paper containing formulae only (no text), a formula sheet (given), calculator.

Time allowed: 1 hour. Each question has equal marks.

Advice: Don't cram your answers into too small a space – try to spread out your answers.

1a). A particle has the following potential energy as a function of distance along the *x*-axis (assume V_0 is positive):

$$V(x) = V_0 \qquad x < -2a$$
$$V(x) = -V_0(x+a)/a \qquad -2a < x < -a$$
$$V(x) = 0 \qquad -a < x < a$$
$$V(x) = V_0(x-a)/a \qquad a < x < 2a$$
$$V(x) = V_0 \qquad x > 2a$$

i) Sketch the potential energy curve.
ii) What is the force exerted on the particle in all 5 regions of space?

b). The energy eigenfunctions for a single particle in an infinite square well with walls at $x = 0$ and $x = a$ are given by:

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a} \qquad 0 < x < a$$

$$\psi_n(x) = 0 \qquad x < 0,\ x > a$$

It is a general fact that solutions of the time independent Schrödinger equation corresponding to different energies are orthogonal to each other, i.e. for different *m* and *n*:

$$\int_{-\infty}^{\infty} \psi_m(x)\psi_n(x)dx = 0$$

Verify this for the two solutions $\psi_1(x)$ and $\psi_2(x)$.

2. a) What is meant by the term *expectation value*?
b) What is the expectation value of the potential energy, *V*, for a simple harmonic oscillator of mass *m* and force constant *k* in its ground state, described by the energy eigenfunction:

$$\psi_0(x) = \left(\frac{km}{\pi^2 \hbar^2}\right)^{1/8} e^{-\left(\frac{km}{\hbar^2}\right)^{\frac{1}{2}} \frac{x^2}{2}}$$

c) Is this expectation value less than or more than the total energy of this state. Is this what you would expect?

3. a) Identify the quantum numbers required to specify an electronic state of a hydrogen atom and give the relationships between the possible values of these quantum numbers. What do these quantum numbers represent?
b) Give the functional form of, and sketch the radial probability density function, $P(r)$, for an electron in the $2s$ state and for an electron in the $2p_0$ state. Explain how you obtained each part of each sketch.
c) Calculate the average distance between the electron and the proton in the $2p_0$ state. Is this greater or less than the most probable value of the distance?
d) The hydrogen atom emission spectrum results from transitions from higher to lower energy eigenstates. The transitions to each level of a particular *n* form a series, named (in order of increasing wavelength, starting with transitions to $n = 1$) Lyman, Balmer, Paschen, Brackett etc. The Lyman series does not overlap the Balmer series. Which of these series is the first to overlap its neighbour? Demonstrate how you got your answer.