Physics 203H Midterm, March 8th, 2005

Allowed: 1 single-sided sheet of 8½×11" paper containing your notes, formula sheet (given), calculator.

Time allowed: 55 mins. Each question has equal marks. Your top nine marks count.

Advice: Don't cram your answers into too small a space – try to spread out your answers.

1. What is the quantum mechanical operator for the following physical observables?
a) the linear momentum of a particle of mass m free to move only along the y-axis?
b) the kinetic energy of a particle of mass m, free to move in 3-D space?
c) the kinetic energy of two particles of mass m_1 and m_2, free to move in 3-D space?
d) the total energy operator of a hydrogen atom, neglecting spin?

2. Describe in a few lines what is meant by the following terms:
a) *the Hamiltonian operator*
b) *an eigenvalue equation*
c) *the time-independent Schrödinger equation*
d) *an expectation value*

3. Describe qualitatively the general form of an energy eigenfunction for a one-electron atom, such as H, or He^+, neglecting spin. Upon how many quantum numbers does it depend, what are they, and what do they physically represent?

4. Sketch a Grotian diagram of all states of the hydrogen atom (neglecting spin) up to $n = 6$. What are the energies of all these states in eV, and how many states are there in total?

5. Over what range of n is the spacing between hydrogenic energy levels with principal quantum numbers n and n+1 less than 0.01 eV?

6. Give the functional form of, and sketch, the radial probability density function $P(r)$ for an electron in a hydrogen atom in state $3d_{+2}$.

7. Derive the $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ operators in Cartesian coordinates.

8. Consider a hydrogen atom in a state described by the $2p_x$ wave function:

$$\psi_{2p_x} = \frac{1}{\sqrt{2}}\left(\psi_{2p,+1} + \psi_{2p,-1}\right)$$

Show that this is not an eigenfunction of $\hat{L}_z$. What is the significance of this?

9. Find all possible values of the total angular momentum quantum numbers J and M_J that can occur in a system with two sources of angular momentum, with angular momentum quantum numbers $J_1 = 3$ and $J_2 = 2$.

10. Explain what is meant by a *singlet state* and a *triplet state* in a helium atom and how these relate to the symmetry of the wave function under exchange of electron labels.