Final Exam: Tuesday Apr 7, 2020

(Delivered online in 2020, with slightly different format, due to COVID-19 pandemic)

Allowed: Formula sheet (given), calculator, 2 hours

Answer questions **1-12** on rough paper. Take your time. Then email answers to **rayfshiell@trentu.ca**.

For all except the first question:

If correct answer in **one** = **4** points; correct answer in **two** = **2** points; correct answer in **three** = **0.5** points

1) [no marks for this question] Which of the following are the first two digits of your six-digit student number (dropping any leading zeros)?

A. 64 **B.** 63 **C.** 62 **D.** 61 **E.** 60 **F.** 59

2) Suppose a multiparticle system has total orbital angular quantum number, ℓ_T, given by the first digit of your student number, and total spin angular quantum number, s_T, given by the second digit of your student number (excluding any leading zeros). How many distinct values of j_T result from adding these two angular momenta together?

A. 1 **B.** 3 **C.** 5 **D.** 7 **E.** 9 **F.** 11

Qu's 3 – 6: Consider the electronic state of atomic hydrogen, neglecting spin, with $(n,\ell,m_\ell)=(2,1,0)$.

3) With the help of the formula sheet, which of the following is the correct mathematical form of the spatial wave function, $\psi(r,\theta,\phi)$ for this state, neglecting any normalization constant?

A. $\left(6-\frac{r}{a_0}\right)r\,e^{-r/3a_0}\cos\theta$ **B.** $re^{-r/2a_0}\cos\theta$ **C.** $\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$ **D.** $\left(6-\frac{r}{a_0}\right)r\,e^{-r/3a_0}\sin\theta e^{i\phi}$ **E.** $re^{-r/2a_0}\sin\theta e^{i\phi}$

4) For this state what is the eigenvalue of the total energy operator, $\hat{H}$? (N.B. You may either do a long eigenvalue calculation, or instead a quicker calculation with the appropriate formula from the formula sheet).

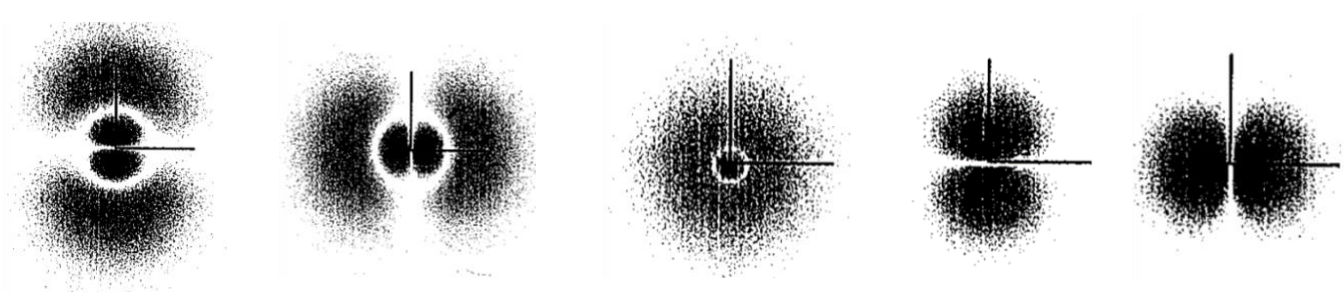
A. -2.41×10^{-19} J **B.** -5.42×10^{-19} J **C.** -2.17×10^{-18} J **D.** $\sqrt{2}\hbar$ **E.** $1\hbar$

5) Which of the following experimental measurements on many atoms in this state will theoretically give <u>zero uncertainty</u> from these experiments?

A. Orbital angular momentum magnitude and orbital angular momentum projection on *z*-axis only
B. Orbital angular momentum magnitude only
C. Total energy and orbital angular momentum magnitude only
D. Total energy, orbital angular momentum magnitude, orbital angular momentum projection on *z*-axis
E. Total energy and orbital angular momentum projection on *z*-axis only

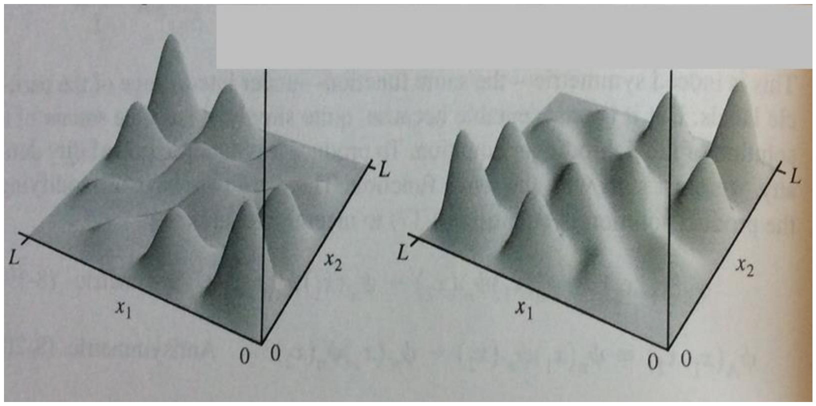
6) Which of the following probability density plots from Harris (i.e. density plots of $|\psi(r,\theta,\phi)|^2$) best resembles this state? (here the *z*-axis is in the plane of, and points up, the page).

A. **B.** **C.** **D.** **E.**



Qu's 7 – 8: Two spinless identical particles in an infinite well of length L occupy the $n'=3$ and $n=4$ energy states. The two physically possible wavefunctions are called '*symmetric*' and '*antisymmetric*' wavefunctions, with labels $\psi_S(x_1,x_2)$ and $\psi_A(x_1,x_2)$, respectively.

7) Looking at the plots of corresponding *probability density* below, what, in each case, is plotted?



A. Left = $|\psi_A(x_1,x_2)|^2$, Right = $|\psi_S(x_1,x_2)|^2$

B. Left = $\psi_S(x_1,x_2)$, Right = $\psi_A(x_1,x_2)$

C. Left = $\psi_A(x_1,x_2)$, Right = $\psi_S(x_1,x_2)$

D. Left = $|\psi_S(x_1,x_2)|^2$, Right = $|\psi_A(x_1,x_2)|^2$

8) Which of the following <u>wavefunctions</u> corresponds to each of these plots?

A.

Left plot $\Rightarrow \frac{\sqrt{2}}{L}\left(\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)+\sin\left(\frac{4\pi x_1}{L}\right)\sin\left(\frac{3\pi x_2}{L}\right)\right)$

Right plot $\Rightarrow \frac{\sqrt{2}}{L}\left(\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)-\sin\left(\frac{4\pi x_1}{L}\right)\sin\left(\frac{3\pi x_2}{L}\right)\right)$

B.

Left plot $\Rightarrow \frac{\sqrt{2}}{L}\left(\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)-\sin\left(\frac{4\pi x_1}{L}\right)\sin\left(\frac{3\pi x_2}{L}\right)\right)$

Right plot $\Rightarrow \frac{\sqrt{2}}{L}\left(\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)+\sin\left(\frac{4\pi x_1}{L}\right)\sin\left(\frac{3\pi x_2}{L}\right)\right)$

C.

Left plot $\Rightarrow \frac{\sqrt{2}}{L}\left(\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)+\sin\left(\frac{4\pi x_2}{L}\right)\sin\left(\frac{3\pi x_1}{L}\right)\right)$

Right plot $\Rightarrow \frac{\sqrt{2}}{L}\left(\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)-\sin\left(\frac{4\pi x_2}{L}\right)\sin\left(\frac{3\pi x_1}{L}\right)\right)$

D.

Left plot $\Rightarrow \frac{\sqrt{2}}{L}\left(\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)-\sin\left(\frac{4\pi x_2}{L}\right)\sin\left(\frac{3\pi x_1}{L}\right)\right)$

Right plot $\Rightarrow \frac{\sqrt{2}}{L}\left(\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)+\sin\left(\frac{4\pi x_2}{L}\right)\sin\left(\frac{3\pi x_1}{L}\right)\right)$

Qu's 9 – 10: Suppose a homonuclear diatomic molecule has a spring constant of $45\ \text{Nm}^{-1}$, and is composed from two atoms, each with mass number *A*, lying along the *x*-axis. Here, *A* is given by the last two digits of your student number [e.g. if your student number were 765432 then *A* would be 32].

9) Which of the following statements is most accurate about the $\sigma 2p$ orbital for this molecule?

A. If two electrons are placed into this molecular orbital then they must have the same spin value, m_s.
B. This molecular orbital can hold up to four electrons.
C. This molecular orbital stems from 2p atomic orbitals, and so can hold up to $2\times 3=6$ electrons.
D. This molecular orbital can be formed from the summation of one $2p_x$ atomic orbital from each atom, where these atomic orbitals lie perpendicular to the internuclear axis.
E. This molecular orbital can be formed from the summation of one $2p_x$ atomic orbital from each atom, where each of these atomic orbitals lie along the internuclear axis.

10) In which energy region lies the energy spacing, ΔE_{vib}, between the lowest vibrational state (labelled by $n=0$) and the first-excited vibrational state (labelled by $n=1$)?

A. $\Delta E_{\text{vib}} < 20$ meV **B.** 20 meV < $\Delta E_{\text{vib}} < 40$ meV **C.** 40 meV < $\Delta E_{\text{vib}} < 60$ meV
D. 60 meV < $\Delta E_{\text{vib}} < 80$ meV **E.** $\Delta E_{\text{vib}} > 80$ meV

Qu's 11 – 12) The ordering of the first few orbitals within the nucleus derived from the shell model is as follows: $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$.

11) Nuclei with so-called *magic numbers* of neutrons or protons have high levels of stability associated with closed subshells. Given this, which value below is a possible magic number for protons **or** neutrons?

A. 4 **B.** 8 **C.** 10 **D.** 12 **E.** 18

12) What is the total angular momentum quantum number of the ${}^{17}_{8}\text{O}$ nucleus (called its nuclear *spin*)?

A. $\frac{1}{2}$ **B.** $\frac{3}{2}$ **C.** $\frac{5}{2}$ **D.** $\frac{7}{2}$ **E.** $\frac{9}{2}$