

Final Exam: Saturday Apr 13, 2019

Name: _____

Allowed: Formula sheet (given), calculator, 3 hours

PART 1 – answer questions **1-10** in your exam booklet, **then** use your scratch card

One scratch worth **4** points; two scratches **2** points; three scratches **0.5** point (total for part 1 = 50%)

PART 2 – answer questions **11, 12 and 13** in the exam booklet provided (total for part 2 = 50%)

Qu's 1 – 4: Consider the quantum state of atomic hydrogen, neglecting spin, with $(n, \ell, m_\ell) = (3, 1, 0)$.

1) With the help of the formula sheet, which of the following is the correct mathematical form of the spatial wave function, $\psi(r, \theta, \phi)$ for this state, neglecting any normalization constant?

- A. $\left(6 - \frac{r}{a_0}\right) r e^{-r/3a_0} \cos \theta$ B. $r e^{-r/2a_0} \cos \theta$ C. $\left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$ D. $\left(6 - \frac{r}{a_0}\right) r e^{-r/3a_0} \sin \theta e^{i\phi}$ E. $r e^{-r/2a_0} \sin \theta e^{i\phi}$

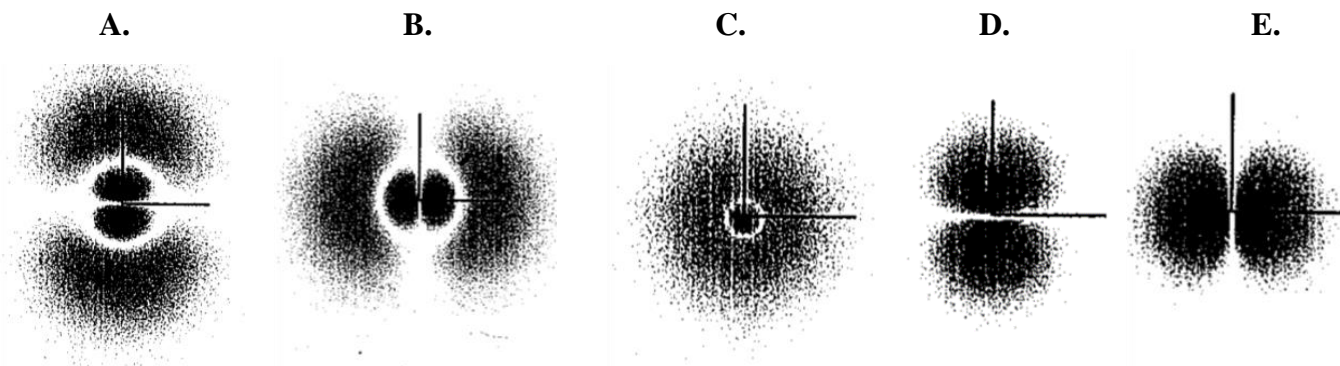
2) For this state what is the eigenvalue of the total energy operator, \hat{H} ? (N.B. You may either do a long eigenvalue calculation, or instead a quicker calculation with the appropriate formula from the formula sheet).

- A. -2.41×10^{-19} J B. -5.42×10^{-19} J C. -2.17×10^{-18} J D. $\sqrt{2}\hbar$ E. $1\hbar$

3) Which of the following experimental measurements on many atoms in this state will theoretically give zero uncertainty from these experiments?

- A. Orbital angular momentum magnitude and orbital angular momentum projection on z -axis only
 B. Orbital angular momentum magnitude only
 C. Total energy and orbital angular momentum magnitude only
 D. Total energy, orbital angular momentum magnitude, orbital angular momentum projection on z -axis
 E. Total energy and orbital angular momentum projection on z -axis only

4) Which of the following probability density plots (i.e. density plots of $|\psi(r, \theta, \phi)|^2$) best resembles this state? (N.B. the z -axis points vertically up the page).



Qu's 5 – 8: Consider an excited helium atom with one electron in the 1s orbital, and a second electron in an orbital with orbital angular momentum quantum number of 2.

5) Using the usual notation of $^{2s_r+1} \ell_T j_r$ for atomic energy levels, which are possible for this atom?

- A. $^1D_2, ^3D_2, ^3D_1, ^3D_0$ B. $^1P_1, ^3D_3, ^3D_2, ^3D_1$ C. $^1P_2, ^3P_2, ^3P_1, ^3P_0$
 D. $^1D_2, ^3D_3, ^3D_2, ^3D_1$ E. $^1P_2, ^3P_3, ^3P_2, ^3P_1$

6) Which of the following expressions best represents the *spin* part of the two-electron wave function when the atom is in the *singlet* ($s_r = 0$) level?

- A. $\uparrow_1 \uparrow_2$ B. $\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2$ C. $\uparrow_1 \downarrow_2 + \downarrow_1 \uparrow_2$ D. $\uparrow_1 \downarrow_2$ E. $\downarrow_1 \downarrow_2$

7) Which of the following best describes the *exchange symmetry* (i.e. symmetry with respect to exchange of electron labels) of the spatial part, the spin part, and the total wave function for the *singlet* level?

- A. The spin part is antisymmetric; the spatial part is antisymmetric; the total wave function is symmetric
 B. The spin part is antisymmetric; the spatial part is symmetric; the total wave function is symmetric
 C. The spin part is symmetric; the spatial part is symmetric; the total wave function is antisymmetric
 D. The spin part is symmetric; the spatial part is antisymmetric; the total wave function is antisymmetric
 E. The spin part is antisymmetric; the spatial part is symmetric; the total wave function is antisymmetric

8) Are the two electrons likely to be further apart or closer together in the singlet level compared to a triplet level – and what effect, if any, does this have on their relative energies?

- A. In a singlet level the electrons are closer together than in triplet levels, so singlets have a lower energy
 B. In a singlet level the electrons are further apart than in triplet levels, so singlets have a higher energy
 C. In a singlet level the electrons are closer together than in triplet levels, so singlets have a higher energy
 D. In a singlet level the electrons are further apart than in triplet levels, so singlets have a lower energy
 E. In a singlet level the electrons are closer together than in triplet levels, with no effect on the energy

Qu's 9 – 10: Consider the following data from the course textbook:

Mass of a neutron:	1.008665u
Atomic mass of ^1_1H :	1.007825u
Atomic mass of ^3_2He :	3.016029u
Atomic mass of ^4_2He :	4.002603u

9) What is the binding energy of ^4_2He ?

- A. 30.3 MeV B. 8.00 MeV C. 28.3 MeV D. 19.8 MeV E. 20.6 MeV

10) What is the energy required to remove one *neutron* from ^4_2He ?

- A. 30.3 MeV B. 8.00 MeV C. 28.3 MeV D. 19.8 MeV E. 20.6 MeV

PART II – answer all three questions in your exam booklet

11. Two non-interacting particles, assumed spinless and *distinguishable*, occupy two different energy states ($n = 2$ and $n' = 3$) in a 1-D infinite well of length L . One possible *product wave function* is:

$$\psi(x_1, x_2) = \frac{2}{L} \sin \frac{2\pi x_1}{L} \sin \frac{3\pi x_2}{L}$$

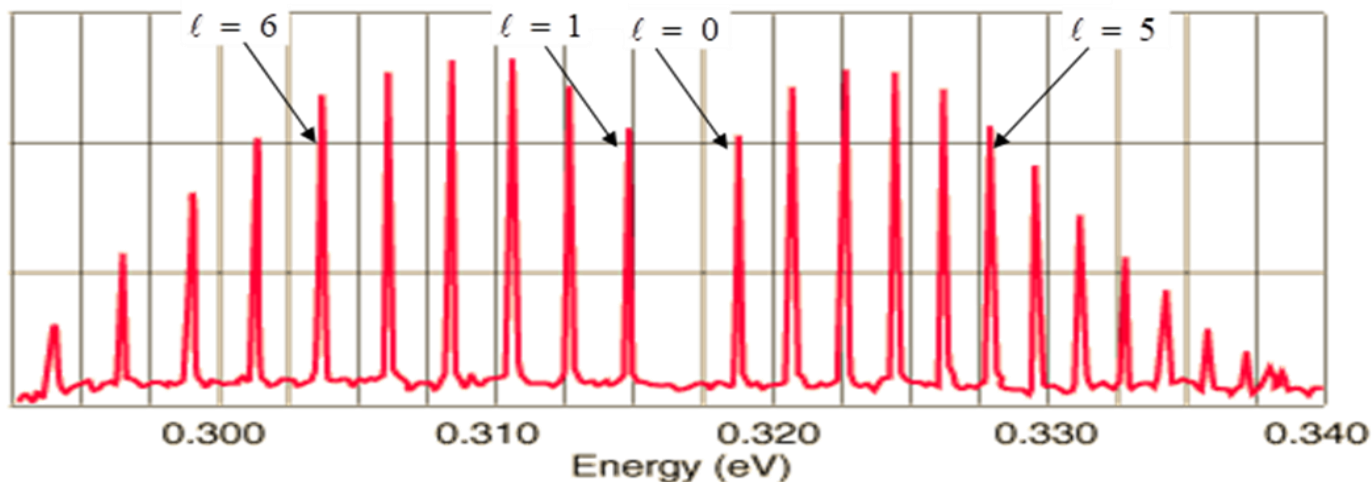
a) Show explicitly that with the normalization constant of $2/L$ this product wave function is *normalized*.

b) Now assume these two particles are instead *identical* and have spin. How should this product spatial wave function be adapted, and which two possible wave functions result? Given that the normalization constant for each of these two possible wave functions is $\sqrt{2}/L$, calculate the probability for each wave function for *both* particles being found between $x=0$ and $x=L/6$.

$$\left[\text{Note: } \int_0^{L/6} \sin \frac{2\pi x}{L} \sin \frac{3\pi x}{L} dx = \frac{L}{5\pi} \right]$$

12. a) Sketch some vibration-rotation energy levels of a diatomic molecule, labelling them with appropriate quantum numbers and indicate an approximate energy scale. Show, with an arrow between states, some *absorption transitions* that satisfy the selection rules $\Delta n = \pm 1$, $\Delta \ell = \pm 1$.

b) Explain in a few sentences the absorption spectrum of H^{79}Br shown below, where a few absorption lines have been labelled by the value of ℓ in the lower state. Estimate from the graph i) the H^{79}Br bond length, and ii) the effective spring constant between the two atoms. [Rough estimates only are required provided you show your reasoning; you may, but do not need to, use a ruler for this question]



<http://hyperphysics.phy-astr.gsu.edu/hbase/molecule/vibrot3.html>

13. The semiempirical binding energy formula takes the form (with energy values in MeV):

$$BE = 15.8A - 17.8A^{2/3} - 0.71 \frac{Z(Z-1)}{A^{1/3}} - 23.7 \frac{(N-Z)^2}{A}$$

a) Use this formula and also the data given for Qu's 9 & 10 to find the mass of hypothetical nucleus ${}_{119}^{288}\text{X}$.

b) Write down the paths that would depict this nucleus undergoing: α -decay, β^+ decay, and β^- decay.

c) Using the semiempirical binding energy formula for all resulting heavy nuclei, and also the data given for Qu's 9 & 10 above, determine the energy release (if any) from each of these three decay paths.