Final Exam: Wednesday April 13, 2016

Allowed: **Time – three hours**
Formula sheet (given)
Calculator

Each question carries equal marks. Answer <u>three</u> out of the four questions.

For each problem <u>briefly justify</u> the equations you choose with a simple term/phrase – randomly writing down a set of equations from your formula sheet will not earn any marks.

Do not cram your answers into too small a space – try to spread out your answers.

1 a) Consider the 2s electron in a hydrogen atom.

i) Carefully sketch, with an explanation, the radial probability density, $P(r)$, for this electron. Show by substitution into the appropriate formula that one peak of this function is located at $r \approx 0.76\,\mathrm{a}_0$, and one peak is at $r \approx 5.24\ \mathrm{a}_0$.

ii) Estimate the probability that this electron may be found with a distance from the nucleus in the range $\Delta r = 0.01\ \mathrm{a}_0$ at $5.24\ \mathrm{a}_0$.

b) Suppose two excited energy levels in an atom are separated by a very small energy difference δE. When atoms in these energy levels decay to the ground state they emit light with nearly identical wavelengths (i.e. approximately the same λ, but different by $\Delta\lambda$). Show that:

$$\Delta\lambda \approx \frac{\lambda^2}{hc}\delta E\ .$$

2. Two particles, assumed initially to be distinguishable from each other and also to be spinless, occupy two different energy states ($n = 2$ and $n' = 3$) in a 1-D infinite well of length L. The *product wave function* for the case where particle 1 is in state $n = 2$ and particle 2 is in state $n' = 3$ is given by:

$$\psi(1,2) = \frac{2}{L}\sin\frac{2\pi x_1}{L}\sin\frac{3\pi x_2}{L}$$

a) Show explicitly that with the normalization constant of $2/L$ this product wave function is *normalized*.

b) Now assume the two particles are instead *identical*. How should this product wave function be adapted, and which two possible wave functions result? Given that the normalization constant of each of these two possible wave functions is $\sqrt{2}/L$, calculate the probability in each case of <u>both</u> particles being found between 0 and $L/6$.

c) Now assume the two identical particles also have *spin, with* $s = 1/2$. Write down using a convenient and clear notation each of the four possible two-particle wave functions for this system.

$$\left[\text{Note:} \qquad \int_0^{L/6}\sin\frac{2\pi x}{L}\sin\frac{3\pi x}{L}\,dx = \frac{L}{5\pi}\qquad\right]$$

3. Consider the $H^{19}F$ molecule to be modelled by two masses connected by a spring of spring constant $965\ \text{Nm}^{-1}$, with average bond length of 0.92 Å.

a) Find the energies in eV of the *three* lowest rotational energy states (labelled by ℓ) in *each* of the *two* lowest vibrational stacks (labelled by n), and sketch and label all six energy states.

b) Find the wavelength of light absorbed or emitted in all possible transitions between these six states that satisfy the selection rules $\Delta n = \pm 1,\ \Delta\ell = \pm 1$ and label the transitions on your sketch. In what region of the electromagnetic spectrum do these occur?

4. a) **Five** distinct types of radioactive decay were introduced in this course. Explain each type clearly, and describe in each case what happens to *A* and *Z* for the decay of a generic nucleus ${}_{Z}^{A}X$.

i) Determine the *mass threshold* for β^+ decay; i.e. what is the minimum *atomic* mass difference between the parent and daughter to make β^+ decay energetically possible?

ii) Using the table below, which of the four nuclei ^{13}N, ^{14}N, ^{40}Ca, ^{152}Ho might exhibit β^+ decay?

b) The ordering of the first few orbitals within the nucleus derived from the shell model is:
$1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$. Using the shell model:

i) Deduce the total angular momentum quantum number of the ${}_{8}^{17}$O nucleus (called its nuclear *spin*).

ii) Determine which of the nuclei with $Z \leq 20$ in the table below are likely to be particularly stable, and explain your reasoning.

Element	**Symbol**		**Mass, u**
electron	*e*		0.0005486
neutron	n		1.008665
Element	**Symbol**	***Z***	**Atomic mass, u**
carbon-13	^{13}C	6	13.003355
carbon-14	^{14}C	6	14.003241
nitrogen-13	^{13}N	7	13.005738
nitrogen-14	^{14}N	7	14.003074
potassium-40	^{40}K	19	39.963999
calcium-40	^{40}Ca	20	39.962591
dysprosium-152	^{152}Dy	66	151.924716
holmium-152	^{152}Ho	67	151.931580