Final Exam: Thursday April 16, 2015

Allowed: **Time – three hours**
Formula sheet (given)
Calculator

Each question carries equal marks. Answer three out of the four questions.

For each problem briefly justify the equations you choose with a simple term/phrase – randomly writing down a set of equations from your formula sheet will not earn any marks.

Do not cram your answers into too small a space – try to spread out your answers.

1. Find the form of the electron radial probability density, $P(r)$, for the 3d state of the hydrogen atom and then answer the following questions:

a) By determining its value as $r \to 0$ and $r \to \infty$, and the position of its maximum, carefully sketch $P(r)$ and provide typical units for each axis.

b) Write down, but do **not** calculate, the computation required to find the probability of the electron being found somewhere between $r = 6\mathrm{a}_0$ and $r = 8\mathrm{a}_0$.

c) Estimate the probability of the electron being found within a spherical shell of thickness $0.06\,a_0$ near $r \approx 6\mathrm{a}_0$ and also near $r \approx 8\mathrm{a}_0$.

d) Explain what is meant by the term *expectation value of r*, and determine the value of this quantity.

2. Two particles, assumed initially to be distinguishable from each other and also to be spinless, occupy two different energy states ($n = 2$ and $n' = 3$) in a 1-D infinite well of length L. The *product wave function* for the case where particle 1 is in state $n = 2$ and particle 2 is in state $n' = 3$ is given by:

$$\psi(1,2) = \frac{2}{L}\sin\frac{2\pi x_1}{L}\sin\frac{3\pi x_2}{L}$$

a) Show explicitly that with the normalization constant of $2/L$ this product wave function is *normalized*.

b) Now assume the two particles are instead *identical*. How should this product wave function be adapted, and which two possible wave functions result? Given that the normalization constant of each of these two possible wave functions is $\frac{\sqrt{2}}{L}$, calculate the probability in each case of both particles being found in the left side of the well (i.e. between 0 and $L/2$).

c) Now assume the two identical particles also have *spin, with* $s = 1/2$. Write down using a convenient and clear notation each of the four possible two-particle wave functions for this system.

$$\left[\text{Note:} \qquad \int_0^{L/2} \sin\frac{2\pi x}{L}\sin\frac{3\pi x}{L}\,dx = \frac{2L}{5\pi}\right]$$

3. Consider the $H^{19}F$ molecule to be modelled by two masses connected by a spring of spring constant $965\ \mathrm{Nm}^{-1}$, with average bond length of 0.92 Å.

a) Find the energies in eV of the *three* lowest rotational energy states (labelled by ℓ) in *each* of the *two* lowest vibrational stacks (labelled by n), and sketch and label all six energy states.

b) Find the wavelength of light absorbed or emitted in all possible transitions between these six states that satisfy the selection rules $\Delta n = \pm 1,\ \Delta\ell = \pm 1$ and label the transitions on your sketch. In what region of the electromagnetic spectrum do these occur?

4. a) **Five** distinct types of *radioactive decay* were introduced in this course. Briefly explain each type clearly, and describe in each case what happens to A and Z for the decay of a generic nucleus ${}^{A}_{Z}X$.

b) Consider the following table of some masses:

Element	**Symbol**		**Mass, u**
electron	e		0.0005486
neutron	n		1.008665
Element	**Symbol**	***Z***	**Atomic mass, u**
hydrogen	^{1}H	1	1.007825
helium-4	^{4}He	2	4.002603
carbon-13	^{13}C	6	13.003355
carbon-14	^{14}C	6	14.003241
nitrogen-13	^{13}N	7	13.005738
nitrogen-14	^{14}N	7	14.003074
potassium-40	^{40}K	19	39.963999
calcium-40	^{40}Ca	20	39.962591
lead-206	^{206}Pb	82	205.974440
polonium-210	^{210}Po	84	209.982848

Calculate how much kinetic energy is released in the β^{+} decay of ^{13}N, and also in the α decay of ^{210}Po.