Trent University: PHYS 2620H – Atomic, Molecular and Nuclear Physics: 2011 – 2012

Final Exam: Friday April 13, 2012

Allowed: **Time – three hours**
Formula sheet (given)
Calculator

Each question carries equal marks. Answer three out of the four questions.

For each problem briefly justify the equations you choose with a simple term/phrase – randomly writing down a set of equations from your formula sheet will not earn any marks.

Do not cram your answers into too small a space – try to spread out your answers.

1. a) Find the form of the electron radial probability density, $P(r)$, for the 1s state of the hydrogen atom. By determining appropriate values such as its value as $r \to 0, \infty$ and the position of its maximum, sketch this function. Estimate the probability of a 1s electron being found within a spherical shell of thickness $0.06\, a_0$ at $r \approx 2a_0$ and compare this with an estimate of it being found within $0.06\, a_0$ of the proton.

b) What is the expected angle between $\vec{L}$ and $\vec{S}$ in i) the $3d_{3/2}$ and ii) the $3d_{5/2}$ state of the hydrogen atom?

2. a) Consider an infinite-well potential of length L containing five identical non-interacting particles. What is the minimum possible total energy of this system in each case if the particles have spin angular momentum quantum number: i) $s = 1/2$, ii) $s = 1$ and iii) $s = 3/2$?

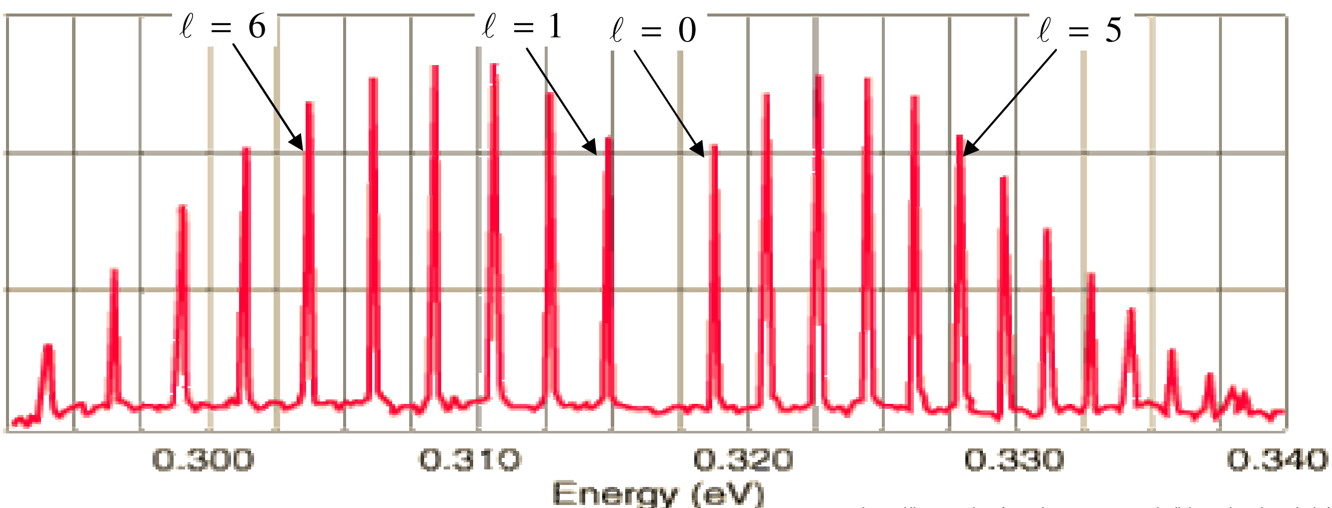
b) Consider two different, perfectly acceptable, wavefunctions for two electrons in an atom, ψ_a and ψ_b:

$$\psi_a(1,2) = \phi_a(\vec{r}_1, \vec{r}_2)\uparrow_1\uparrow_2$$
$$\psi_b(1,2) = \phi_b(\vec{r}_1, \vec{r}_2)\frac{1}{\sqrt{2}}\left(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\right)$$

i) In each case is the spin part of the wavefunction symmetric, antisymmetric or asymmetric, with respect to exchange of the two electrons' labels?
ii) In each case, is the spatial part of the wavefunction symmetric, antisymmetric or asymmetric under particle exchange? Explain why.
iii) For which of the two wavefunctions ψ_a and ψ_b do the electrons have a very low likelihood of both being in a region of space such that they are very close to each other? Explain why this is so.

3. a) Sketch the form of a few of the vibration-rotation energy levels of a diatomic molecule (labelling them with appropriate quantum numbers) and show, with an arrow between states, a set of absorption transitions that satisfy the selection rules $\Delta n = \pm 1,\ \Delta \ell = \pm 1$.

b) Explain in a few sentences the optical absorption spectrum of HBr shown below, where a few of the absorption lines have been labelled by the value of ℓ in the lower state. From the graph estimate i) the HBr bond length, and ii) the effective spring constant between the two atoms. [Rough estimates only are required provided you clearly state where the numerical values you use come from; you may, but do not need to, use a ruler for this question. Note also that bromine has a mass of 79u]



http://hyperphysics.phy-astr.gsu.edu/hbase/molecule/vibrot3.html

4. a) Explain clearly what is meant by the following types of radioactive decay: α, β^-, β^+, *electron capture*, and γ decay. In all cases denote what happens to *A* and *Z* for a generic nucleus ${}^{A}_{Z}X$.

b) Consider the following table of some atomic masses (1 u = 931.5 MeV/c^2):

Element	Symbol	Z	Atomic mass, u
electron	*e*	-1	0.0005486
neutron	n	0	1.008665
hydrogen	^{1}H	1	1.007825
helium-4	^{4}He	2	4.002603
carbon-13	^{13}C	6	13.003355
carbon-14	^{14}C	6	14.003241
nitrogen-13	^{13}N	7	13.005738
nitrogen-14	^{14}N	7	14.003074
potassium-40	^{40}K	19	39.963999
calcium-40	^{40}Ca	20	39.962591
lead-206	^{206}Pb	82	205.974440
polonium-210	^{210}Po	84	209.982848

Determine how much kinetic energy is released in the β^+ decay of ^{13}N, and in the α decay of ^{210}Po.