Final Exam: Wednesday April 13, 2011

Allowed: **Time – three hours**
Formula sheet (given)
Calculator

Each question carries equal marks. Answer <u>three</u> out of the four questions.

For each problem <u>briefly justify</u> the equations you choose with a simple term/phrase – randomly writing down a set of equations from your formula sheet will not earn any marks.

Do not cram your answers into too small a space – try to spread out your answers.

1. a) As clearly as you can, sketch the shape of the electron probability density for the $n = 2, \ell = 1, m_\ell = 0$ state of the hydrogen atom. Explain how you obtained your sketch.
b) On a separate plot sketch the *radial probability density*, $P(r)$.
c) Find the distance from the nucleus that the electron is most likely to be found in the state given in (a).
d) For this state, what is the *expectation value* of this distance, and what is meant by the term *expectation value*?
e) Estimate the probability of the electron being found within $0.06\, a_0$ of the result you calculate in (d).

2. a) Two identical particles in an infinite well occupy the $n = 1$ and $n' = 2$ individual-particle states. Write down the wave functions of the possible symmetric and antisymmetric spatial states, taking the normalization constant of each wave function to be a parameter, *A*.
b) Write down (but do not calculate) the integral that has to be evaluated to find the probability of one particle being located in the region between $x_{1,a}$ and $x_{1,b}$ and the other particle in the region between $x_{2,a}$ and $x_{2,b}$. If these two regions have substantial overlap, which of the two spatial wave functions is likely to give a greater probability of finding the particles in these regions, and why?
c) If the two particles are electrons, what *exchange symmetry* must be satisfied by the *total wave function*? Using each of the two spatial wave functions from part (a), write down two possible total wave functions, including spin.

3. Consider the $^{12}C^{16}O$ molecule to be modelled by two masses connected by a spring with spring constant 1900 Nm^{-1}, and average bond length of 1.12 Å.
a) Calculate the energies in eV of the four lowest rotational states (labelled by ℓ) within <u>each</u> of the two lowest vibrational states (labelled by *n*). Sketch these eight energy states in order of their energy, and label the energy and quantum numbers of each state.
b) On your sketch add one vertical arrow denoting an *allowed transition*, and calculate the energy and wavelength corresponding to this transition. In what region of the electromagnetic spectrum does it lie?
c) The ionization energy of carbon (atomic number 6) is 11.3 eV and that of oxygen (atomic number 8) is 13.6 eV. Provide a sketch demonstrating how the *molecular orbitals* of the CO molecule can be created from the atomic orbitals of each atom and, by providing a set of labels for these molecular orbitals, write down the lowest-energy electron configuration of CO.

4. Consider the following table of atomic masses (1 u = 931.5 MeV/c^2):

Element	Symbol	Z	Atomic mass, u
electron	e	-1	0.0005486
neutron	n	0	1.008665
hydrogen	^{1}H	1	1.007825
carbon-13	^{13}C	6	13.003355
carbon-14	^{14}C	6	14.003241
nitrogen-13	^{13}N	7	13.005738
nitrogen-14	^{14}N	7	14.003074
potassium-40	^{40}K	19	39.963999
calcium-40	^{40}Ca	20	39.962591

The human body contains several radioactive elements, most notably ^{14}C and ^{40}K, which both decay by β^- decay with a similar overall rate of decay ($\approx$4,000 per second per person).

a) Explain carefully what is meant by the term "β^- *decay*", and write down the precise process and resulting particles for both ^{14}C and ^{40}K.
b) Use the table above to calculate the kinetic energy released by each ^{14}C, and each ^{40}K, decay.
c) The ^{13}N nucleus decays by β^+ decay. Write down the precise process and resulting particles for this decay, and calculate the kinetic energy released (hint: carefully count all electron/positron masses).
d) Determine the total *binding energy* in MeV, and also the binding energy per nucleon, of ^{40}Ca, and compare this with the semi-empirical binding energy formula below.

$$BE = c_1 A - c_2 A^{2/3} - c_3 \frac{Z(Z-1)}{A^{1/3}} - c_4 \frac{(Z - A/2)^2}{A}$$

$$c_1 = 15.8\,\text{MeV}, c_2 = 17.8\,\text{MeV}, c_3 = 0.71\,\text{MeV}, c_4 = 23.7\,\text{MeV}$$