Trent: PHYS 2620H – Atomic, Molecular and Nuclear Physics: 2009 – 2010

Final Exam: Thursday April 15, 2010

Allowed: **Time – three hours**
Formula sheet (given)
Calculator

Each question carries equal marks. Answer three out of the four questions.

For each problem briefly explain the equations you choose to use – simply writing down a set of equations from your formula sheet will not earn any marks.

Don't cram your answers into too small a space – try to spread out your answers.

1. A particle of mass m has a potential energy given by $V(x) = \frac{1}{2}kx^2$. It is in the first excited energy state, with wavefunction:

$$\psi(x) = A\,x\exp\left(-\left(\frac{mk}{\hbar^2}\right)^{\frac{1}{2}}\frac{x^2}{2}\right)$$

where A is a normalization constant.
a) Calculate in terms of A the expectation values of x, p and p^2 in this state. (Hint: Use symmetry where possible and explain why).
b) Write down a formula representing the vibrational energies of a HF molecule in terms of an effective spring constant, k, and the masses m_H, m_F of the constituent atoms.
c) Given that the wavelength of the light emitted by HF in a transition from $v = 1 \rightarrow v = 0$ state is 2.4 μm, find the value of k that characterizes the H – F bond. (The atomic mass of F is 19 amu).

2.a) Define what is meant by a *transition dipole moment* for a transition from state ψ_2 to state ψ_1.
b) Calculate the z-component of the transition dipole moment between the $2p_0$ and the 1s state in the hydrogen atom (neglect spin), and also determine the wavelength of this transition.
c) From this value calculate Einstein's B_{21} (with units given by m^3/Js^2), and A_{21} coefficients.
d) What is the spontaneous emission lifetime of the $2p_0$ state in hydrogen?

3. a) Consider putting two electrons into an infinite well of width L, with one electron in the first excited state, and the other in the second excited state. What *exchange symmetry* must be satisfied by these electrons? Write down an acceptable wavefunction for this state (you can treat the normalization constant as a parameter, A, if you wish).
b) The electronic configuration of H_2 in its lowest energy state is $1\sigma_g^2$. Sketch the form of the molecular orbital $1\sigma_g$ and explain how it can be formed from a linear combination of atomic orbitals. The spin part of the electronic wavefunction for this state is given by

$$\chi = \frac{1}{\sqrt{2}}\left(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\right).$$

State whether χ is a symmetric or antisymmetric wavefunction under exchange of the two electrons and show this explicitly. Given this, what can be said about the exchange symmetry of the *spatial* part of the wavefunction?

4. a) Given the following table of atomic masses:

Element	Symbol	Atomic mass, u
neutron	n	1.008665
hydrogen	^{1}H	1.007825
lithium	^{7}Li	7.016003

Determine the total binding energy in MeV, and also the binding energy per nucleon, of $^{7}_{3}\mathrm{Li}$.

b) Sketch the approximate form of the curve of binding energy per nucleon versus mass number, and put approximate numerical scales on each axis.

c) The semi-empirical mass formula, based on the liquid-drop model of the nucleus, is given below. Discuss the origin of each term in the formula, and explain why each specific term is either positive or negative.

$$M_{nucleus} = Zm_p + Nm_n - c_1 A + c_2 A^{2/3} + c_3 \frac{Z(Z-1)}{A^{1/3}} + c_4 \frac{(Z - A/2)^2}{A}$$